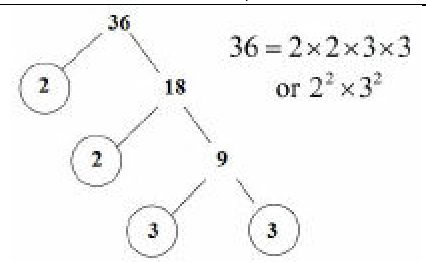
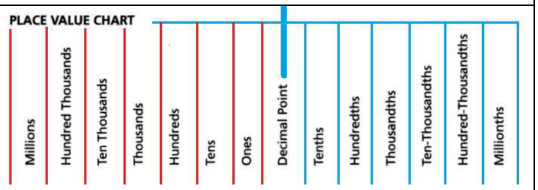
Topic: Factors and Multiples

Topic/Skill	Definition/Tips	Example
1. Multiple	The result of multiplying a number by an integer. The **times tables** of a number.	The first five multiples of 7 are: $7, 14, 21, 28, 35$
2. Factor	A number that **divides exactly** into another number without a remainder. It is useful to write factors in pairs	The factors of 18 are: $1, 2, 3, 6, 9, 18$ The factor pairs of 18 are: $1, 18$ $2, 9$ $3, 6$
3. Lowest Common Multiple (LCM)	The **smallest** number that is in the **times tables** of each of the numbers given.	The LCM of 3, 4 and 5 is 60 because it is the smallest number in the 3, 4 and 5 times tables.
4. Highest Common Factor (HCF)	The **biggest** number that **divides exactly** into two or more numbers.	The HCF of 6 and 9 is 3 because it is the biggest number that divides into 6 and 9 exactly.
5. Prime Number	A number with **exactly two factors**. A number that can only be divided by itself and one. The number **1 is not prime**, as it only has one factor, not two.	The first ten prime numbers are: $2, 3, 5, 7, 11, 13, 17, 19, 23, 29$
6. Prime Factor	A factor which is a prime number.	The prime factors of 18 are: $2, 3$
7. Product of Prime Factors	Finding out which **prime numbers multiply** together to make the **original** number. Use a **prime factor tree.** Also known as 'prime factorisation'.	

Topic: Accuracy

Topic/Skill	Definition/Tips	Example
1. Place Value	The **value** of where a **digit** is within a number.	In 726, the value of the 2 is 20, as it is in the ‘tens’ column.
2. Place Value Columns	The names of the columns that **determine the value of each digit**. The ‘ones’ column is also known as the ‘units’ column.	
3. Rounding	To make a number simpler but keep its value close to what it was. If the **digit to the right** of the rounding digit is **less than 5, round down**. If the **digit to the right** of the rounding digit is **5 or more, round up**.	74 rounded to the nearest ten is 70, because 74 is closer to 70 than 80. 152,879 rounded to the nearest thousand is 153,000.
4. Decimal Place	The **position** of a digit to the **right of a decimal point**.	In the number 0.372, the 7 is in the second decimal place. 0.372 rounded to two decimal places is 0.37, because the 2 tells us to round down. Careful with money - don’t write £27.4, instead write £27.40
5. Significant Figure	The significant figures of a number are the digits which **carry meaning** (ie. are significant) to the size of the number. The **first significant figure** of a number **cannot be zero**. In a number with a decimal, trailing zeros are not significant.	In the number 0.00821, the first significant figure is the 8. In the number 2.740, the 0 is not a significant figure. 0.00821 rounded to 2 significant figures is 0.0082. 19357 rounded to 3 significant figures is 19400. We need to include the two zeros at the end to keep the digits in the same place value columns.
6. Truncation	A method of approximating a decimal number by **dropping all decimal places** past a certain point **without rounding**.	3.14159265… can be truncated to 3.1415 (note that if it had been rounded, it would become 3.1416)
7. Error Interval	A **range of values** that a number could have taken before being rounded or truncated. An error interval is written using inequalities, with a **lower bound** and an **upper bound**.	0.6 has been rounded to 1 decimal place. The error interval is: $0.55 \leq x < 0.65$ The lower bound is 0.55 The upper bound is 0.65

	Note that the lower bound inequality can be 'equal to', but the upper bound cannot be 'equal to'.	
8. Estimate	To find something **close to the correct answer**.	An estimate for the height of a man is 1.8 metres.
9. Approximation	When using approximations to estimate the solution to a calculation, **round each number in the calculation to 1 significant figure**. ≈ means 'approximately equal to'	$\frac{348+692}{0.526} \approx \frac{300+700}{0.5} = 2000$ 'Note that dividing by 0.5 is the same as multiplying by 2'
10. Rational Number	A number of the form $\frac{p}{q}$, where $\boldsymbol{p}$ **and** $\boldsymbol{q}$ **are integers** and $\boldsymbol{q \neq 0}$. A number that cannot be written in this form is called an 'irrational' number	$\frac{4}{9}, 6, -\frac{1}{3}, \sqrt{25}$ are examples of rational numbers. $\pi, \sqrt{2}$ are examples of an irrational numbers.
11. Surd	The **irrational number** that is a **root of a positive integer,** whose value cannot be determined exactly. Surds have **infinite non-recurring decimals**.	$\sqrt{2}$ is a surd because it is a root which cannot be determined exactly. $\sqrt{2} = 1.41421356\ldots$ which never repeats.
12. Rules of Surds	$\boldsymbol{\sqrt{ab} = \sqrt{a} \times \sqrt{b}}$ $\boldsymbol{\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}}$ $\boldsymbol{a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}}$ $\boldsymbol{\sqrt{a} \times \sqrt{a} = a}$	$\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$ $\sqrt{\frac{25}{36}} = \frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$ $2\sqrt{5} + 7\sqrt{5} = 9\sqrt{5}$ $\sqrt{7} \times \sqrt{7} = 7$
13. Rationalise a Denominator	The process of rewriting a fraction so that the **denominator contains only rational numbers**.	$\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{6}}{2}$ $\frac{6}{3+\sqrt{7}} = \frac{6(3-\sqrt{7})}{(3+\sqrt{7})(3-\sqrt{7})}$ $= \frac{18-6\sqrt{7}}{9-7}$ $= \frac{18-6\sqrt{7}}{2} = 9 - 3\sqrt{7}$

Topic: Indices

Topic/Skill	Definition/Tips	Example
1. Square Number	The number you get when you **multiply a number by itself**.	**1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225…** $9^2 = 9 \times 9 = 81$
2. Square Root	The **number you multiply by itself** to get another number. The reverse process of squaring a number.	$\sqrt{36} = 6$ because $6 \times 6 = 36$
3. Solutions to $x^2 = \ldots$	**Equations** involving **squares** have **two solutions**, one **positive** and one **negative**.	Solve $x^2 = 25$ $x = 5 \text{ or } x = -5$ This can also be written as $x = \pm 5$
4. Cube Number	The number you get when you **multiply a number by itself and itself again**.	1, 8, 27, 64, 125… $2^3 = 2 \times 2 \times 2 = 8$
5. Cube Root	The **number you multiply by itself and itself again** to get another number. The reverse process of cubing a number.	$\sqrt[3]{125} = 5$ because $5 \times 5 \times 5 = 125$
6. Powers of…	The powers of a number are that **number raised to various powers**.	The powers of 3 are: $3^1 = 3$ $3^2 = 9$ $3^3 = 27$ $3^4 = 81$ etc.
7. Multiplication Index Law	When **multiplying** with the same base (number or letter), **add the powers**. $\boldsymbol{a^m \times a^n = a^{m+n}}$	$7^5 \times 7^3 = 7^8$ $a^{12} \times a = a^{13}$ $4x^5 \times 2x^8 = 8x^{13}$
8. Division Index Law	When **dividing** with the same base (number or letter), **subtract the powers**. $\boldsymbol{a^m \div a^n = a^{m-n}}$	$15^7 \div 15^4 = 15^3$ $x^9 \div x^2 = x^7$ $20a^{11} \div 5a^3 = 4a^8$
9. Brackets Index Laws	When raising a power to another power, multiply the powers together. $\boldsymbol{(a^m)^n = a^{mn}}$	$(y^2)^5 = y^{10}$ $(6^3)^4 = 6^{12}$ $(5x^6)^3 = 125x^{18}$
10. Notable Powers	$\boldsymbol{p = p^1}$ $\boldsymbol{p^0 = 1}$	$99999^0 = 1$
11. Negative Powers	A negative power performs the reciprocal. $\boldsymbol{a^{-m} = \frac{1}{a^m}}$	$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
12. Fractional Powers	The denominator of a fractional power acts as a 'root'. The numerator of a fractional power acts as a normal power. $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$	$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$ $\left(\frac{25}{16}\right)^{\frac{3}{2}} = \left(\frac{\sqrt{25}}{\sqrt{16}}\right)^3 = \left(\frac{5}{4}\right)^3 = \frac{125}{64}$

Topic/Skill	Definition/Tips	Example
1. Standard Form	$\boldsymbol{A \times 10^b}$ $where\ \boldsymbol{1 \leq A < 10}, \quad \boldsymbol{b = integer}$	$8400 = 8.4 \times 10^3$ $0.00036 = 3.6 \times 10^{-4}$
2. Multiplying or Dividing with Standard Form	Multiply: **Multiply the numbers** and **add the powers**. Divide: **Divide the numbers** and **subtract the powers**.	$(1.2 \times 10^3) \times (4 \times 10^6) = 8.8 \times 10^9$ $(4.5 \times 10^5) \div (3 \times 10^2) = 1.5 \times 10^3$
3. Adding or Subtracting with Standard Form	**Convert** in to **ordinary** numbers, **calculate** and then **convert back** in to standard form	$2.7 \times 10^4 + 4.6 \times 10^3$ $= 27000 + 4600 = 31600$ $= 3.16 \times 10^4$

Topic: Summarising Data

<table>
<tr><th>Topic/Skill</th><th>Definition/Tips</th><th>Example</th></tr>
<tr><td>1. Types of Data</td><td>Qualitative Data – non-numerical data
Quantitative Data – numerical data

Continuous Data – data that can take any numerical value within a given range.
Discrete Data – data that can take only specific values within a given range.</td><td>Qualitative Data – eye colour, gender etc.

Continuous Data – weight, voltage etc.

Discrete Data – number of children, shoe size etc.</td></tr>
<tr><td>2. Grouped Data</td><td>Data that has been bundled in to categories.

Seen in grouped frequency tables, histograms, cumulative frequency etc.</td><td><table><tr><th>Foot length, l, (cm)</th><th>Number of children</th></tr><tr><td>$10 \leqslant l < 12$</td><td>5</td></tr><tr><td>$12 \leqslant l < 17$</td><td>53</td></tr></table></td></tr>
<tr><td>3. Primary /Secondary Data</td><td>Primary Data – collected yourself for a specific purpose.

Secondary Data – collected by someone else for another purpose.</td><td>Primary Data – data collected by a student for their own research project.

Secondary Data – Census data used to analyse link between education and earnings.</td></tr>
<tr><td>4. Mean</td><td>Add up the values and divide by how many values there are.</td><td>The mean of 3, 4, 7, 6, 0, 4, 6 is
$\frac{3+4+7+6+0+4+6}{7} = 5$</td></tr>
<tr><td>5. Mean from a Table</td><td>1. Find the midpoints (if necessary)
2. Multiply Frequency by values or midpoints
3. Add up these values
4. Divide this total by the Total Frequency

If grouped data is used, the answer will be an estimate.</td><td><table><tr><th>Height in cm</th><th>Frequency</th><th>Midpoint</th><th>F × M</th></tr><tr><td>$0 < h \leq 10$</td><td>8</td><td>5</td><td>8×5=40</td></tr><tr><td>$10 < h \leq 30$</td><td>10</td><td>20</td><td>10×20=200</td></tr><tr><td>$30 < h \leq 40$</td><td>6</td><td>35</td><td>6×35=210</td></tr><tr><td>Total</td><td>24</td><td>Ignore!</td><td>450</td></tr></table>Estimated Mean height: 450 ÷ 24 = 18.75cm</td></tr>
<tr><td>6. Median Value</td><td>The middle value.

Put the data in order and find the middle one.
If there are two middle values, find the number half way between them by adding them together and dividing by 2.</td><td>Find the median of: 4, 5, 2, 3, 6, 7, 6

Ordered: 2, 3, 4, 5, 6, 6, 7

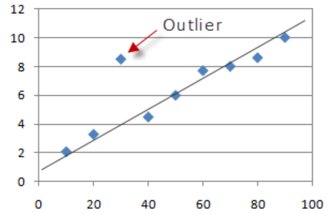
Median = 5</td></tr>
<tr><td>7. Median from a Table</td><td>Use the formula $\frac{(n+1)}{2}$ to find the position of the median.

n is the total frequency.</td><td>If the total frequency is 15, the median will be the $\left(\frac{15+1}{2}\right) = 8th$ position</td></tr>
<tr><td>8. Mode /Modal Value</td><td>Most frequent/common.

Can have more than one mode (called bi-modal or multi-modal) or no mode (if all values appear once)</td><td>Find the mode: 4, 5, 2, 3, 6, 4, 7, 8, 4

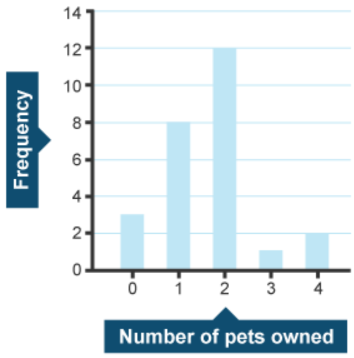
Mode = 4</td></tr>
<tr><td>9. Range</td><td>Highest value subtract the Smallest value</td><td>Find the range: 3, 31, 26, 102, 37, 97.

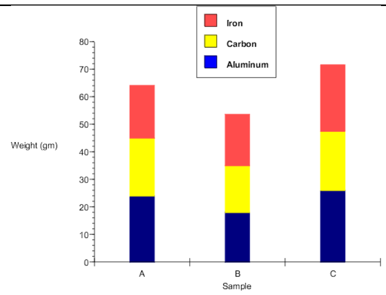
Range = 102-3 = 99</td></tr>
</table>

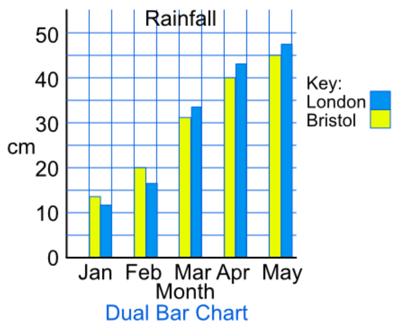
	Range is a 'measure of spread'. The smaller the range the more consistent the data.	
10. Outlier	A value that '**lies outside**' most of the other values in a set of data. An outlier is **much smaller or much larger** than the other values in a set of data.	
11. Lower Quartile	**Divides** the **bottom half** of the data into **two halves**. **LQ** = $\boldsymbol{Q_1 = \frac{(n+1)}{4} th}$ **value**	Find the lower quartile of: 2, **3**, 4, 5, 6, 6, 7 $Q_1 = \frac{(7+1)}{4} = 2nd$ value → 3
12. Lower Quartile	**Divides** the **top half** of the data into **two halves**. **UQ** = $\boldsymbol{Q_3 = \frac{3(n+1)}{4} th}$ **value**	Find the upper quartile of: 2, 3, 4, 5, 6, **6**, 7 $Q_3 = \frac{3(7+1)}{4} = 6th$ value → 6
13. Interquartile Range	The **difference** between the **upper quartile and lower quartile.** $\boldsymbol{IQR = Q_3 - Q_1}$ The **smaller** the **interquartile range**, the **more consistent** the data.	Find the IQR of: 2, 3, 4, 5, 6, 6, 7 $IQR = Q_3 - Q_1 = 6 - 3 = 3$

Topic: Representing Data

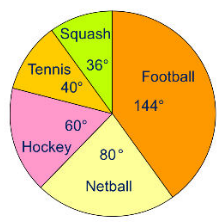
<table>
<tr><th>Topic/Skill</th><th>Definition/Tips</th><th>Example</th></tr>
<tr><td>1. Frequency Table</td><td>A record of how often each value in a set of data occurs.</td><td>
<table>
<tr><th>Number of marks</th><th>Tally marks</th><th>Frequency</th></tr>
<tr><td>1</td><td>卌 ||</td><td>7</td></tr>
<tr><td>2</td><td>卌</td><td>5</td></tr>
<tr><td>3</td><td>卌 |</td><td>6</td></tr>
<tr><td>4</td><td>卌</td><td>5</td></tr>
<tr><td>5</td><td>|||</td><td>3</td></tr>
<tr><td>Total</td><td></td><td>26</td></tr>
</table>
</td></tr>
<tr><td>2. Bar Chart</td><td>Represents data as vertical blocks.

$x-axis$ shows the type of data
$y-axis$ shows the frequency for each type of data
Each bar should be the same width
There should be gaps between each bar
Remember to label each axis.</td><td>
</td></tr>
<tr><td>3. Types of Bar Chart</td><td>Compound/Composite Bar Charts show data stacked on top of each other.

Comparative/Dual Bar Charts show data side by side.</td><td>


</td></tr>
<tr><td>4. Pie Chart</td><td>Used for showing how data breaks down into its constituent parts.

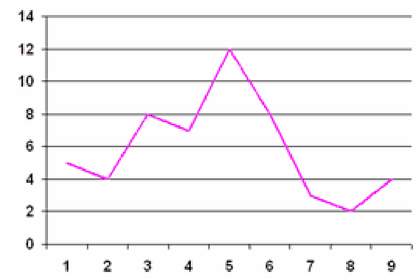
When drawing a pie chart, divide 360° by the total frequency. This will tell you how many degrees to use for the frequency of each category.

Remember to label the category that each sector in the pie chart represents.</td><td>

If there are 40 people in a survey, then each person will be worth 360÷40=9° of the pie chart.</td></tr>
</table>

<table>
<tr>
<td>5. Pictogram</td>
<td>Uses pictures or symbols to show the value of the data.

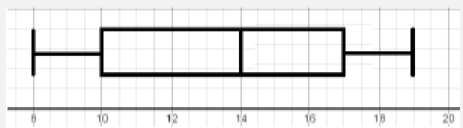
A pictogram must have a key.</td>
<td>
</td>
</tr>
<tr>
<td>6. Line Graph</td>
<td>A graph that uses points connected by straight lines to show how data changes in values.

This can be used for time series data, which is a series of data points spaced over uniform time intervals in time order.</td>
<td>
</td>
</tr>
<tr>
<td>7. Two Way Tables</td>
<td>A table that organises data around two categories.

Fill out the information step by step using the information given.

Make sure all the totals add up for all columns and rows.</td>
<td>Question: Complete the 2 way table below.
<table>
<tr><th></th><th>Left Handed</th><th>Right Handed</th><th>Total</th></tr>
<tr><td>Boys</td><td>10</td><td></td><td>58</td></tr>
<tr><td>Girls</td><td></td><td></td><td></td></tr>
<tr><td>Total</td><td></td><td>84</td><td>100</td></tr>
</table>
Answer: Step 1, fill out the easy parts (the totals)
<table>
<tr><th></th><th>Left Handed</th><th>Right Handed</th><th>Total</th></tr>
<tr><td>Boys</td><td>10</td><td>48</td><td>58</td></tr>
<tr><td>Girls</td><td></td><td></td><td>42</td></tr>
<tr><td>Total</td><td>16</td><td>84</td><td>100</td></tr>
</table>
Answer: Step 2, fill out the remaining parts
<table>
<tr><th></th><th>Left Handed</th><th>Right Handed</th><th>Total</th></tr>
<tr><td>Boys</td><td>10</td><td>48</td><td>58</td></tr>
<tr><td>Girls</td><td>6</td><td>36</td><td>42</td></tr>
<tr><td>Total</td><td>16</td><td>84</td><td>100</td></tr>
</table></td>
</tr>
<tr>
<td>8. Box Plots</td>
<td>The minimum, lower quartile, median, upper quartile and maximum are shown on a box plot.

A box plot can be drawn independently or from a cumulative frequency diagram.</td>
<td>Students sit a maths test. The highest score is 19, the lowest score is 8, the median is 14, the lower quartile is 10 and the upper quartile is 17. Draw a box plot to represent this information.

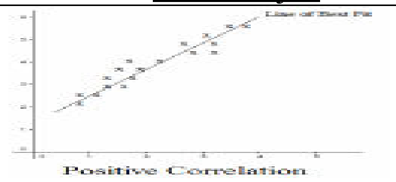
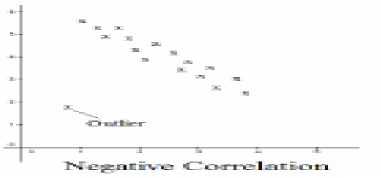
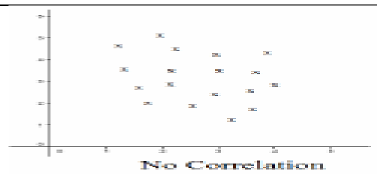
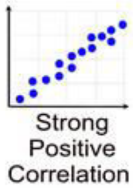
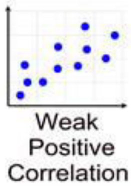
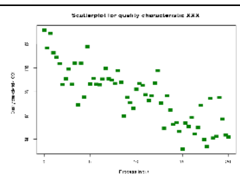
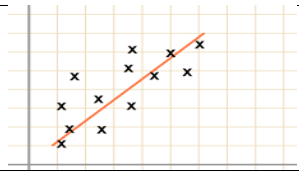
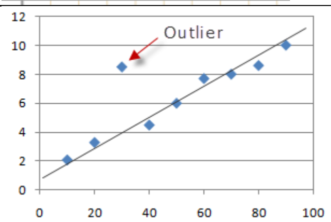

</td>
</tr>
<tr>
<td>9. Comparing Box Plots</td>
<td>Write two sentences.
1. Compare the averages using the medians for two sets of data.
2. Compare the spread of the data using the range or IQR for two sets of data.

The <u>smaller</u> the range/IQR, the <u>more consistent</u> the data.

You must compare box plots in the context of the problem.</td>
<td>‘On average, students in class A were more successful on the test than class B because their median score was higher.’

‘Students in class B were more consistent than class A in their test scores as their IQR was smaller.’</td>
</tr>
</table>

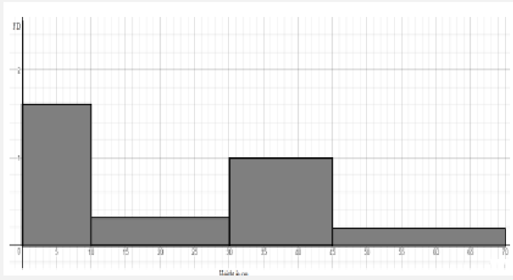
Topic: Scatter Graphs

Topic/Skill	Definition/Tips	Example
1. Correlation	Correlation between two sets of data means they are **connected** in some way.	There is correlation between temperature and the number of ice creams sold.
2. Causality	When one variable **influences** another variable.	The more hours you work at a particular job (paid hourly), the higher your income <u>from that job</u> will be.
3. Positive Correlation	As one value **increases** the other value **increases**.	
4. Negative Correlation	As one value **increases** the other value **decreases**.	
5. No Correlation	There is **no linear relationship** between the two.	
6. Strong Correlation	When two sets of data are **closely linked**.	
7. Weak Correlation	When two sets of data have correlation, but are **not closely linked**.	
8. Scatter Graph	A graph in which values of **two variables** are plotted along two axes to **compare** them and see if there is any **connection** between them.	
9. Line of Best Fit	A **straight line** that **best represents the data** on a scatter graph.	
10. Outlier	A value that 'lies outside' most of the other values in a set of data. An outlier is **much smaller or much larger** than the other values in a set of data.	

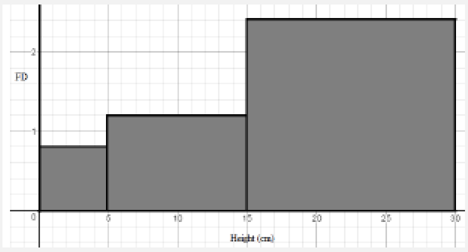
<table>
<tr><th>Topic/Skill</th><th>Definition/Tips</th><th>Example</th></tr>
<tr><td>1. Histograms</td><td>A visual way to display frequency data using bars.

Bars can be unequal in width.

Histograms show frequency density on the y-axis, not frequency.

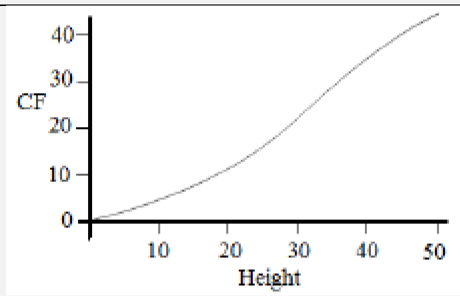
$$\boldsymbol{Frequency\ Density} = \frac{\boldsymbol{Frequency}}{\boldsymbol{Class\ Width}}$$
<table><tr><th>Height(cm)</th><th>Frequency</th></tr><tr><td>$0 < h \leq 10$</td><td>8</td></tr><tr><td>$10 < h \leq 30$</td><td>6</td></tr><tr><td>$30 < h \leq 45$</td><td>15</td></tr><tr><td>$45 < h \leq 70$</td><td>5</td></tr></table></td><td><table><tr><th>Frequency Density (FD)</th></tr><tr><td>$8 \div 5 = 1.6$</td></tr><tr><td>$6 \div 20 = 0.3$</td></tr><tr><td>$15 \div 15 = 1$</td></tr><tr><td>$5 \div 25 = 0.2$</td></tr></table>
</td></tr>
<tr><td>2. Interpreting Histograms</td><td>The area of the bar is proportional to the frequency of that class interval.

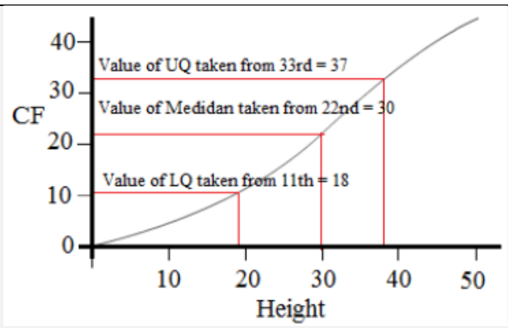
$$\boldsymbol{Frequency} = \boldsymbol{Freq\ Density} \times \boldsymbol{Class\ Width}$$</td><td>A histogram shows information about the heights of a number of plants. 4 plants were less than 5cm tall. Find the number of plants more than 5cm tall.



Above 5cm:
1.2 x 10 + 2.4 x 15 = 12 + 36 = 48</td></tr>
<tr><td>3. Cumulative Frequency</td><td>Cumulative Frequency is a running total.

<table><tr><th>Age</th><th>Frequency</th></tr><tr><td>$0 < a \leq 10$</td><td>15</td></tr><tr><td>$10 < a \leq 40$</td><td>35</td></tr><tr><td>$40 < a \leq 50$</td><td>10</td></tr></table></td><td><table><tr><th>Cumulative Frequency</th></tr><tr><td>15</td></tr><tr><td>$15 + 35 = 50$</td></tr><tr><td>$50 + 10 = 60$</td></tr></table></td></tr>
<tr><td>4. Cumulative Frequency Diagram</td><td>A cumulative frequency diagram is a curve that goes up. It looks a little like a stretched-out S shape.

Plot the cumulative frequencies at the end-point of each interval.</td><td>
</td></tr>
</table>

5. Quartiles from Cumulative Frequency Diagram	**Lower Quartile** (Q1): **25%** of the data is less than the lower quartile. **Median** (Q2): **50%** of the data is less than the median. **Upper Quartile** (Q3): **75%** of the data is less than the upper quartile. **Interquartile Range (IQR)**: represents the **middle 50%** of the data.	 $IQR = 37 - 18 = 19$
6. Hypothesis	**A statement that might be true, which can be tested.**	Hypothesis: 'Large dogs are better at catching tennis balls than small dogs'. We can test this hypothesis by having hundreds of different sized dogs try to catch tennis balls.