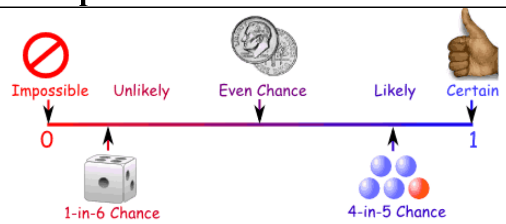
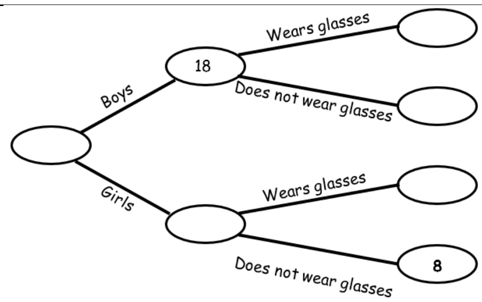
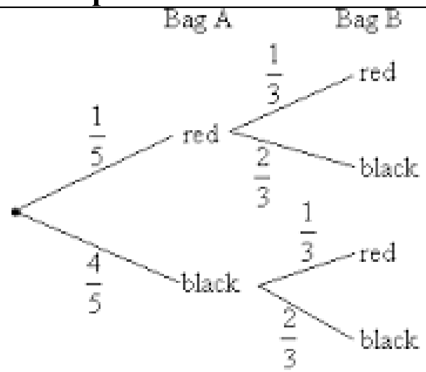
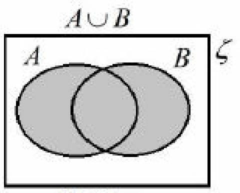
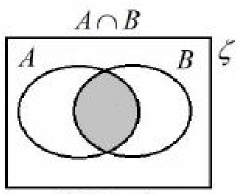
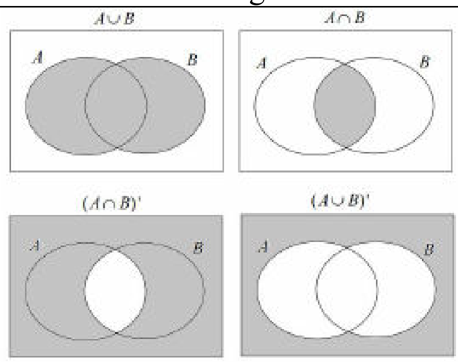
Topic/Skill	Definition/Tips	Example
1. Probability	The **likelihood/chance** of something happening. Is expressed as a number **between 0 (impossible) and 1 (certain)**. Can be expressed as a fraction, decimal, percentage or in words (likely, unlikely, even chance etc.)	
2. Probability Notation	**P(A)** refers to the **probability that event A will occur**.	P(Red Queen) refers to the probability of picking a Red Queen from a pack of cards.
3. Theoretical Probability	$\dfrac{\textit{Number of Favourable Outcomes}}{\textit{Total Number of Possible Outcomes}}$	Probability of rolling a 4 on a fair 6-sided die $= \frac{1}{6}$.
4. Relative Frequency	$\dfrac{\textit{Number of Successful Trials}}{\textit{Total Number of Trials}}$	A coin is flipped 50 times and lands on Tails 29 times. The relative frequency of getting Tails $= \frac{29}{50}$.
5. Expected Outcomes	To find the number of expected outcomes, **multiply** the **probability** by the **number of trials**.	The probability that a football team wins is 0.2 How many games would you expect them to win out of 40? $0.2 \times 40 = 8\ games$
6. Exhaustive	Outcomes are **exhaustive** if they **cover the entire range of possible outcomes**. The **probabilities** of an **exhaustive** set of outcomes **adds up to 1**.	When rolling a six-sided die, the outcomes 1, 2, 3, 4, 5 and 6 are exhaustive, because they cover all the possible outcomes.
7. Mutually Exclusive	Events are mutually exclusive if they **cannot happen at the same time**. The **probabilities** of an exhaustive set of **mutually exclusive** events **adds up to 1**.	Examples of mutually exclusive events: - Turning left and right - Heads and Tails on a coin Examples of non mutually exclusive events: - King and Hearts from a deck of cards, because you can pick the King of Hearts
8. Frequency Tree	A diagram showing how information is categorised into various categories. The **numbers** at the ends of branches tells us how often something happened (**frequency**).	

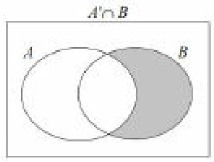
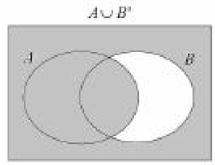
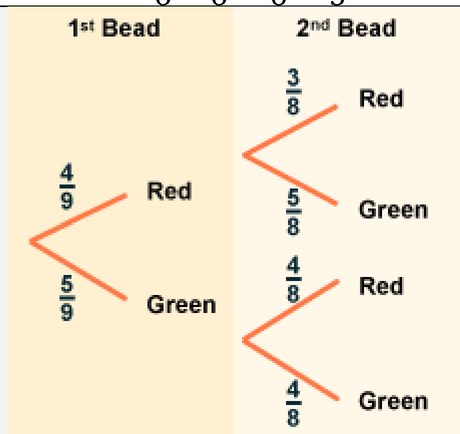
<table>
<tr><td></td><td>The lines connected the numbers are called branches.</td><td></td></tr>
<tr><td>9. Sample Space</td><td>The set of all possible outcomes of an experiment.</td><td><table>
<tr><td>+</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td></tr>
<tr><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr>
</table></td></tr>
<tr><td>10. Sample</td><td>A sample is a small selection of items from a population.

A sample is biased if individuals or groups from the population are not represented in the sample.</td><td>A sample could be selecting 10 students from a year group at school.</td></tr>
<tr><td>11. Sample Size</td><td>The larger a sample size, the closer those probabilities will be to the true probability.</td><td>A sample size of 100 gives a more reliable result than a sample size of 10.</td></tr>
</table>

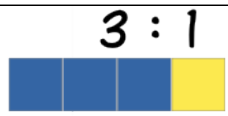
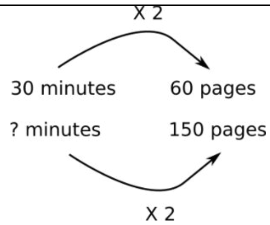
Topic/Skill	Definition/Tips	Example
1. Combination	A collection of things, where the **order does not matter**.	How many combinations of two ingredients can you make with apple, banana and cherry? Apple, Banana Apple, Cherry Banana, Cherry 3 combinations
2. Permutation	A collection of things, where the **order does matter**.	You want to visit the homes of three friends, Alex (A), Betty (B) and Chandra (C) but haven't decided the order. What choices do you have? ABC ACB BAC BCA CAB CBA
3. Permutations with Repetition	When something has n different types, there are **$\boldsymbol{n}$ choices each time**. Choosing r of something that has n different types, the permutations are: $n \times n \times \ldots (r\ times) = \boldsymbol{n^r}$	How many permutations are there for a three-number combination lock? 10 numbers to choose from $\{1, 2, \ldots .10\}$ and we choose 3 of them → $10 \times 10 \times 10 = 10^3 = 1000$ permutations.
4. Permutations without Repetition	We have to **reduce the number of available choices each time**. One you have chosen something, you cannot choose it again.	How many ways can you order 4 numbered balls? $4 \times 3 \times 2 \times 1 = 24$
5. Factorial	The factorial symbol '!' means to multiply a series of descending integers to 1. Note: $0! = 1$	$4! = 4 \times 3 \times 2 \times 1 = 24$
6. Product Rule for Counting	If there are **$\boldsymbol{x}$ ways of doing something** and **$\boldsymbol{y}$ ways of doing something else**, then there are **$\boldsymbol{xy}$ ways of performing both**.	To choose one of $\{A, B, C\}$ and one of $\{X, Y\}$ means to choose one of $\{AX, AY, BX, BY, CX, CY\}$ The rule says that there are $3 \times 2 = 6$ choices.

Topic: Probability (Trees and Venns)

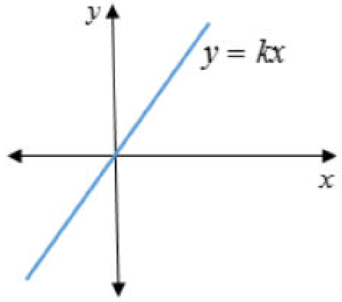
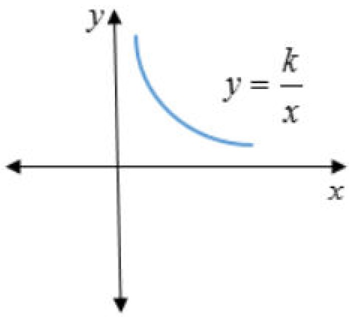
Topic/Skill	Definition/Tips	Example
1. Tree Diagrams	Tree diagrams show **all the possible outcomes** of an event and calculate their probabilities. **All branches must add up to 1 when adding downwards.** This is because the **probability of something not happening** is **1 minus the probability that it does happen**. **Multiply** going **across** a tree diagram. **Add** going **down** a tree diagram.	
2. Independent Events	The outcome of a **previous event does not influence/affect the outcome of a second event**.	An example of independent events could be replacing a counter in a bag after picking it.
3. Dependent Events	The outcome of a **previous event does influence/affect the outcome of a second event**.	An example of dependent events could be not replacing a counter in a bag after picking it. 'Without replacement'
4. Probability Notation	**P(A)** refers to the **probability that event A will occur**. **P(A')** refers to the **probability that event A will not occur**. **P(A ∪ B)** refers to the **probability that event A or B or both will occur.** **P(A ∩ B)** refers to the **probability that both events A and B will occur.**	P(Red Queen) refers to the probability of picking a Red Queen from a pack of cards. P(Blue') refers to the probability that you do not pick Blue. P(Blonde ∪ Right Handed) refers to the probability that you pick someone who is Blonde or Right Handed or both. P(Blonde ∩ Right Handed) refers to the probability that you pick someone who is both Blonde and Right Handed.
5. Venn Diagrams	A Venn Diagram shows the **relationship between a group of different things** and how they overlap. You may be asked to shade Venn Diagrams as shown below and to the right. The Union 'A or B or Both' The Intersection 'A and B'	

		 
6. Venn Diagram Notation	∈ means '**element of a set**' (a value in the set) { } means the collection of values in the set. ξ means the '**universal set**' (all the values to consider in the question) **A' means 'not in set A' (called complement)** **A ∪ B means 'A or B or both' (called Union)** **A ∩ B means 'A and B (called Intersection)**	Set A is the even numbers less than 10. A = {2, 4, 6, 8} Set B is the prime numbers less than 10. B = {2, 3, 5, 7} A ∪ B = {2, 3, 4, 5, 6, 7, 8} A ∩ B = {2}
7. AND rule for Probability	When two events, A and B, are **independent**: $\boldsymbol{P(A \text{ and } B) = P(A) \times P(B)}$	What is the probability of rolling a 4 and flipping a Tails? $P(4 \text{ and } Tails) = P(4) \times P(Tails)$ $= \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$
8. OR rule for Probability	When two events, A and B, are **mutually exclusive**: $\boldsymbol{P(A \text{ or } B) = P(A) + P(B)}$	What is the probability of rolling a 2 or rolling a 5? $P(2 \text{ or } 5) = P(2) + P(5)$ $= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$
9. Conditional Probability	The probability of an event A happening, **given that** event B has already happened. With conditional probability, check if the numbers on the second branches of a tree diagram changes. For example, if you have 4 red beads in a bag of 9 beads and pick a red bead on the first pick, then there will be 3 red beads left out of 8 beads on the second pick.	

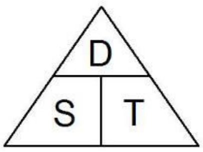
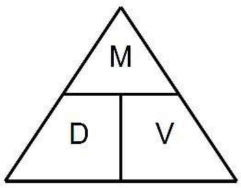
Topic: Ratio

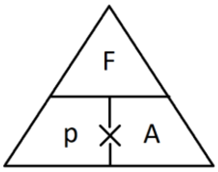
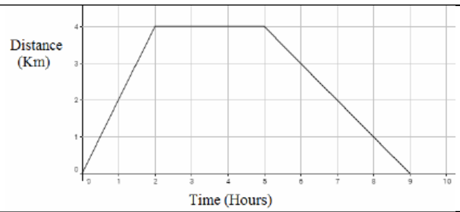
Topic/Skill	Definition/Tips	Example
1. Ratio	Ratio compares the size of **one part** to **another part**. Written using the ':' symbol.	
2. Proportion	Proportion compares the size of **one part** to the size of the **whole**. Usually written as a fraction.	In a class with 13 boys and 9 girls, the proportion of boys is $\frac{13}{22}$ and the proportion of girls is $\frac{9}{22}$
3. Simplifying Ratios	**Divide** all parts of the ratio by a **common factor**.	5 : 10 = 1 : 2 (divide both by 5) 14 : 21 = 2 : 3 (divide both by 7)
4. Ratios in the form 1 : n or n : 1	**Divide** both parts of the ratio by one of the numbers to make **one part equal 1**.	5 : 7 = 1 : $\frac{7}{5}$ in the form 1 : n 5 : 7 = $\frac{5}{7}$: 1 in the form n : 1
5. Sharing in a Ratio	**1. Add** the total parts of the ratio. **2. Divide** the amount to be shared by this value to find the value of one part. **3. Multiply** this value by each part of the ratio. Use only if you **know the total**.	Share £60 in the ratio 3 : 2 : 1. 3 + 2 + 1 = 6 60 ÷ 6 = 10 3 x 10 = 30, 2 x 10 = 20, 1 x 10 = 10 £30 : £20 : £10
6. Proportional Reasoning	Comparing two things using **multiplicative reasoning** and applying this to a new situation. Identify one multiplicative link and use this to find missing quantities.	
7. Unitary Method	Finding the **value of a single unit** and then finding the necessary value by **multiplying** the single unit value.	3 cakes require 450g of sugar to make. Find how much sugar is needed to make 5 cakes. 3 cakes = 450g So 1 cake = 150g (÷ by 3) So 5 cakes = 750 g (x by 5)
8. Ratio already shared	Find what **one part** of the ratio is worth using the **unitary method**.	Money was shared in the ratio 3:2:5 between Ann, Bob and Cat. Given that Bob had £16, found out the total amount of money shared. £16 = 2 parts So £8 = 1 part 3 + 2 + 5 = 10 parts, so 8 x 10 = £80
9. Best Buys	Find the **unit cost** by **dividing** the **price by the quantity**. The **lowest** number is the best value.	8 cakes for £1.28 → 16p each (÷by 8) 13 cakes for £2.05 → 15.8p each (÷by 13) Pack of 13 cakes is best value.

Topic: Proportion

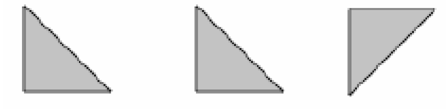
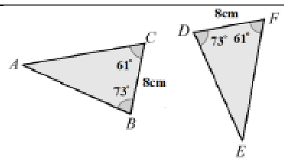
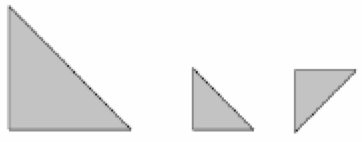
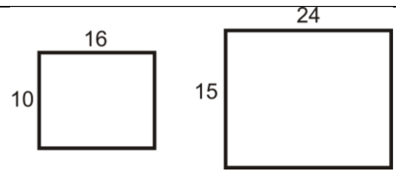
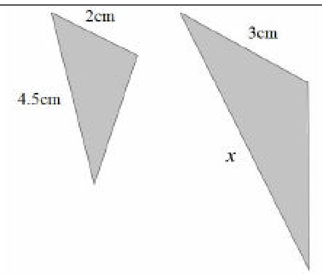
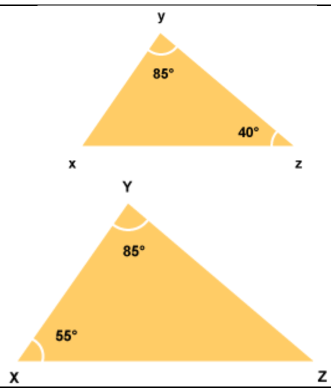
Topic/Skill	Definition/Tips	Example
1. Direct Proportion	If two quantities are in direct proportion, **as one increases**, the **other increases** by the **same percentage**. If y is directly proportional to x, this can be written as $\boldsymbol{y \propto x}$ An equation of the form $\boldsymbol{y = kx}$ represents direct proportion, where k **is the constant of proportionality**.	
2. Inverse Proportion	If two quantities are inversely proportional, **as one increases**, the **other decreases** by the **same percentage**. If y is inversely proportional to x, this can be written as $\boldsymbol{y \propto \frac{1}{x}}$ An equation of the form $\boldsymbol{y = \frac{k}{x}}$ represents inverse proportion.	
3. Using proportionality formulae	**Direct**: **y = kx** or **y** $\propto$ **x** **Inverse**: **y** = $\frac{k}{x}$ or **y** $\propto \frac{1}{x}$ 1. **Solve to find k** using the pair of values in the question. 2. **Rewrite the equation** using the k you have just found. 3. **Substitute the other given value** from the question in to the equation to **find the missing value**.	p is directly proportional to q. When p = 12, q = 4. Find p when q = 20. 1. p = kq 12 = k x 4 so k = 3 2. p = 3q 3. p = 3 x 20 = 60, so p = 60

Topic: Compound Measures

Topic/Skill	Definition/Tips	Example
1. Metric System	A system of measures based on: - the metre for length - the kilogram for mass - the second for time **Length: mm, cm, m, km** **Mass: mg, g, kg** **Volume: ml, cl, l**	$1 kilometres = 1000\ metres$ $1\ metre = 100\ centimetres$ $1\ centimetre = 10\ millimetres$ $1\ kilogram = 1000\ grams$
2. Imperial System	A system of weights and measures originally developed in England, usually based on human quantities **Length: inch, foot, yard, miles** **Mass: lb, ounce, stone** **Volume: pint, gallon**	$1 lb = 16\ ounces$ $1\ foot = 12\ inches$ $1\ gallon = 8\ pints$
3. Metric and Imperial Units	Use the **unitary method** to convert between metric and imperial units.	$5\ miles \approx 8\ kilometres$ $1\ gallon \approx 4.5\ litres$ $2.2\ pounds \approx 1\ kilogram$ $1\ inch = 2.5\ centimetres$
4. Speed, Distance, Time	**Speed = Distance ÷ Time** **Distance = Speed x Time** **Time = Distance ÷ Speed**  Remember the correct units.	Speed = 4mph Time = 2 hours Find the Distance. $D = S \times T = 4 \times 2 = 8\ miles$
5. Density, Mass, Volume	**Density = Mass ÷ Volume** **Mass = Density x Volume** **Volume = Mass ÷ Density**  Remember the correct units.	Density = 8kg/m^3 Mass = 2000g Find the Volume. $V = M \div D = 2 \div 8 = 0.25m^3$
6. Pressure, Force, Area	**Pressure = Force ÷ Area** **Force = Pressure x Area** **Area = Force ÷ Pressure**	Pressure = 10 Pascals Area = 6cm^2 Find the Force

	 Remember the correct units.	$F = P \times A = 10 \times 6 = 60\ N$
7. Distance-Time Graphs	You can find the **speed** from the **gradient** of the line (Distance ÷ Time) The steeper the line, the quicker the speed. A **horizontal** line means the object is not moving (**stationary**).	

Topic: Congruence and Similarity

Topic/Skill	Definition/Tips	Example
1. Congruent Shapes	Shapes are congruent if they are **identical - same shape** and **same size**. Shapes can be rotated or reflected but still be congruent.	
2. Congruent Triangles	4 ways of proving that two triangles are congruent: 1. **SSS** (Side, Side, Side) 2. **RHS** (Right angle, Hypotenuse, Side) 3. **SAS** (Side, Angle, Side) 4. **ASA** (Angle, Side, Angle) or **AAS** <u>ASS does not prove congruency.</u>	 $BC = DF$ $\angle ABC = \angle EDF$ $\angle ACB = \angle EFD$ $\therefore$ The two triangles are congruent by AAS.
3. Similar Shapes	Shapes are similar if they are the **same shape but different sizes**. The proportion of the matching sides must be the same, meaning the ratios of corresponding sides are all equal.	
4. Scale Factor	The **ratio of corresponding sides** of two similar shapes. To find a scale factor, **divide a length** on one shape **by the corresponding length** on a similar shape.	 Scale Factor = $15 \div 10 = 1.5$
5. Finding missing lengths in similar shapes	1. Find the **scale factor**. 2. **Multiply or divide** the corresponding side to find a missing length. If you are finding a missing length on the larger shape you will need to multiply by the scale factor. If you are finding a missing length on the smaller shape you will need to divide by the scale factor.	 Scale Factor = $3 \div 2 = 1.5$ $x = 4.5 \times 1.5 = 6.75cm$
6. Similar Triangles	To show that two triangles are similar, show that: 1. The three sides are in the same proportion 2. Two sides are in the same proportion, and their included angle is the same 3. The three angles are equal	

<table>
<tr><th>Topic/Skill</th><th>Definition/Tips</th><th>Example</th></tr>
<tr><td>1. Exact Values for Angles in Trigonometry</td><td>
<table>
<tr><th></th><th>0°</th><th>30°</th><th>45°</th><th>60°</th><th>90°</th></tr>
<tr><td>sin</td><td>0</td><td>$\frac{1}{2}$</td><td>$\frac{\sqrt{2}}{2}$</td><td>$\frac{\sqrt{3}}{2}$</td><td>1</td></tr>
<tr><td>cos</td><td>1</td><td>$\frac{\sqrt{3}}{2}$</td><td>$\frac{\sqrt{2}}{2}$</td><td>$\frac{1}{2}$</td><td>0</td></tr>
<tr><td>tan</td><td>0</td><td>$\frac{1}{\sqrt{3}}$</td><td>1</td><td>$\sqrt{3}$</td><td>----</td></tr>
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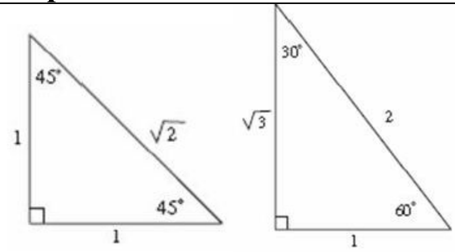
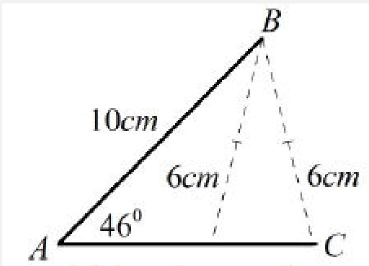
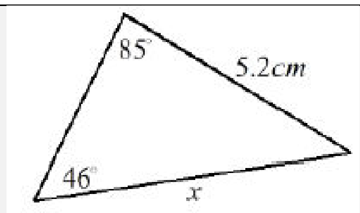
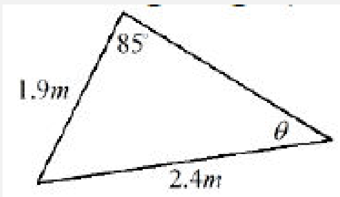
</td><td>


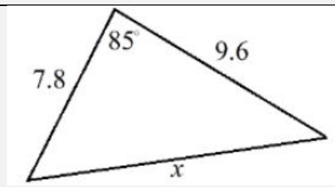
</td></tr>
<tr><td>2. Sine Rule</td><td>
Use with non right angle triangles.
Use when the question involves 2 sides and 2 angles.

For missing side:
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
For missing angle:
$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
There is an ambiguous case (where there are two potential answers)


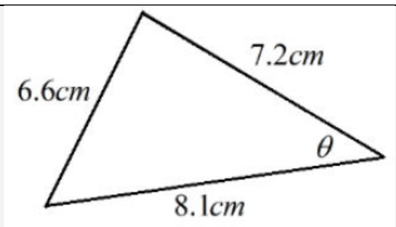
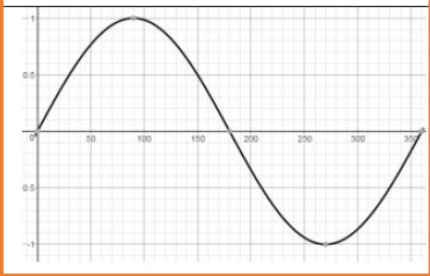
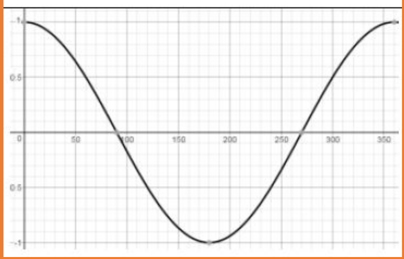
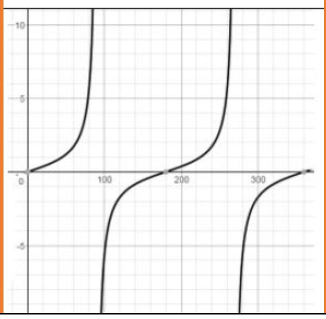
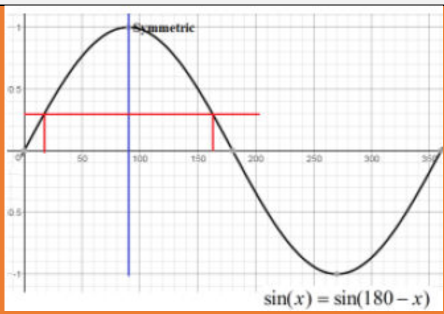
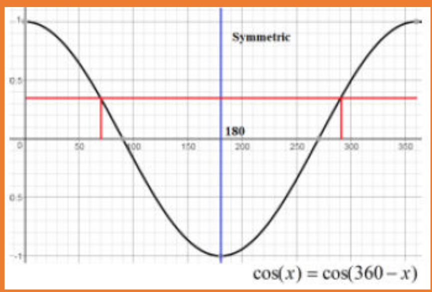
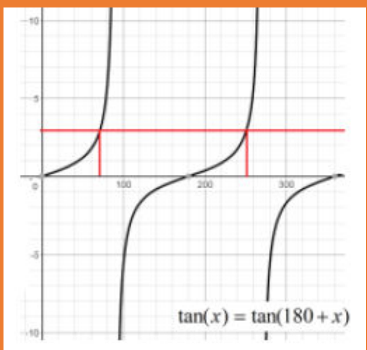
To find the two angles, use sine to find one, and then subtract your answer from 180 to find the other answer.
</td><td>


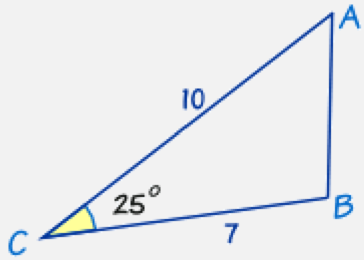
$$\frac{x}{\sin 85} = \frac{5.2}{\sin 46}$$
$$x = \frac{5.2 \times \sin 85}{\sin 46} = 3.75cm$$


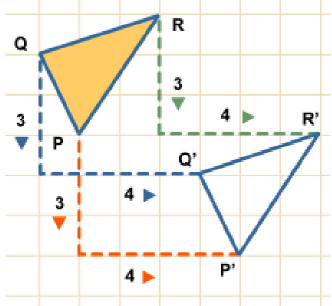
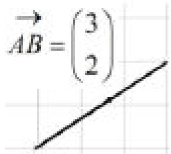
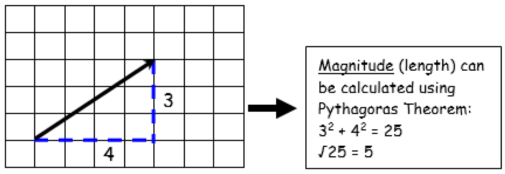
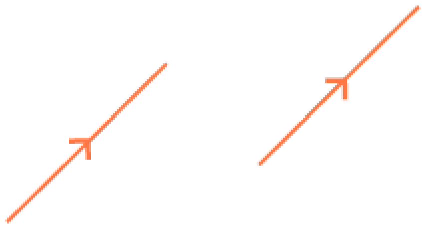
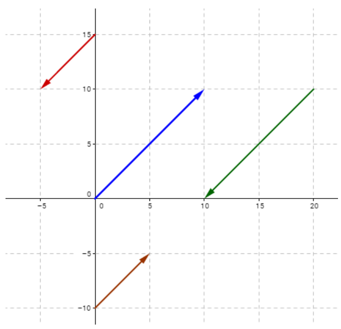
$$\frac{\sin\theta}{1.9} = \frac{\sin 85}{2.4}$$
$$\sin\theta = \frac{1.9 \times \sin 85}{2.4} = 0.789$$
$$\theta = sin^{-1}(0.789) = 52.1°$$
</td></tr>
<tr><td>3. Cosine Rule</td><td>
Use with non right angle triangles.
Use when the question involves 3 sides and 1 angle.

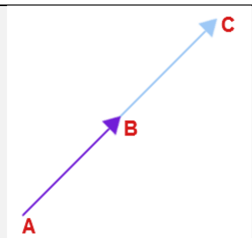
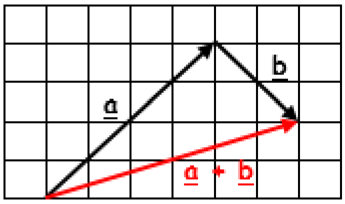
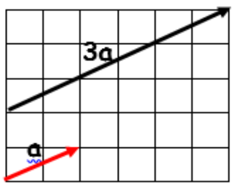
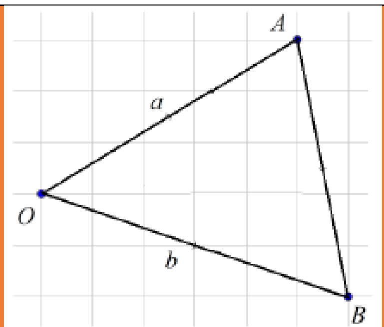
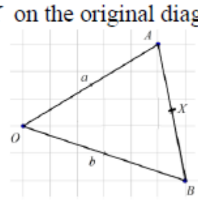
For missing side:
$$\boldsymbol{a^2 = b^2 + c^2 - 2bc\cos A}$$
For missing angle:
$$\boldsymbol{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$$
</td><td>


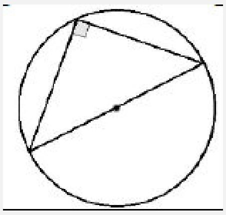
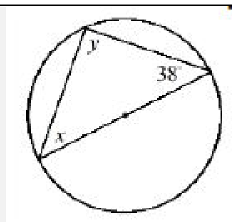
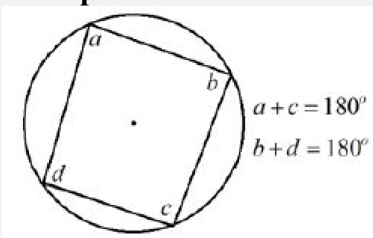
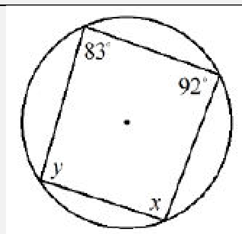
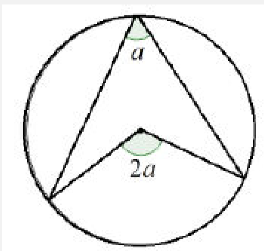
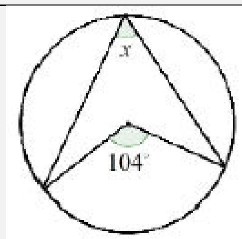
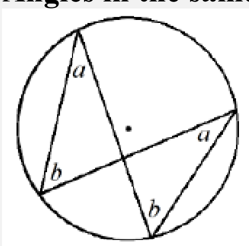
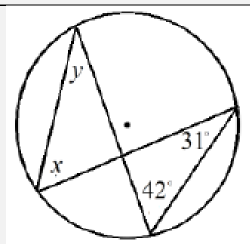
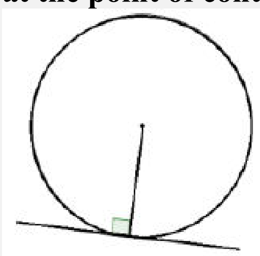
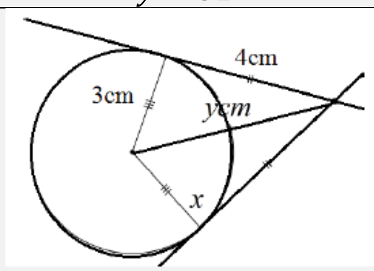
$$x^2 = 9.6^2 + 7.8^2 - (2 \times 9.6 \times 7.8 \times \cos 85)$$
$$x = 11.8$$
</td></tr>
</table>

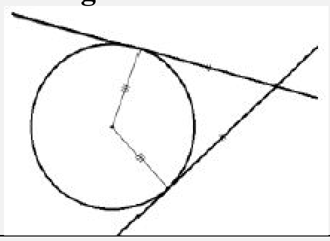
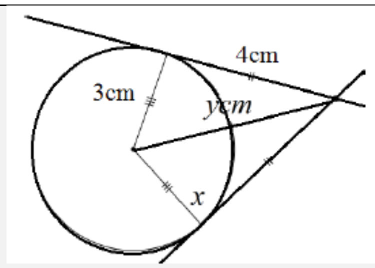
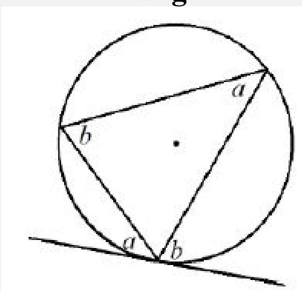
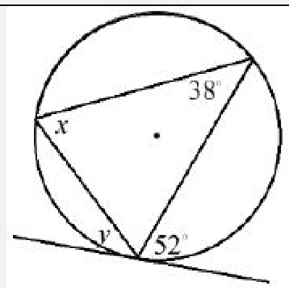
		 $$\cos\theta = \frac{7.2^2 + 8.1^2 - 6.6^2}{2 \times 7.2 \times 8.1}$$ $$\theta = 50.7°$$
4. Graphs of Trigonometric Functions	$y = \sin(x)$ for $0 \le x \le 360^o$  $y = \cos(x)$ for $0 \le x \le 360^o$  $y = \tan(x)$ for $0 \le x \le 360^o$ 	  

5. Area of a Triangle	Use when given the **length of two sides and the included angle**. $$\boldsymbol{Area\ of\ a\ Triangle = \frac{1}{2}ab\sin C}$$	 $$A = \frac{1}{2}ab\sin C$$ $$A = \frac{1}{2} \times 7 \times 10 \times \sin 25$$ $$A = 14.8$$

Topic/Skill	Definition/Tips	Example
1. Translation	**Translate** means to **move a shape**. The shape does not change **size** or **orientation**.	
2. Vector Notation	A vector can be written in 3 ways: **a** or $\overrightarrow{\boldsymbol{AB}}$ or $\begin{pmatrix} \mathbf{1} \\ \mathbf{3} \end{pmatrix}$	
3. Column Vector	In a column vector, the **top** number moves **left (-) or right (+)** and the **bottom** number moves **up (+) or down (-)**	$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ means '2 right, 3 up' $\begin{pmatrix} -1 \\ -5 \end{pmatrix}$ means '1 left, 5 down'
4. Vector	A **vector** is a quantity represented by an arrow with both **direction** and **magnitude**. $\overrightarrow{AB} = -\overrightarrow{BA}$	
5. Magnitude	Magnitude is defined as the **length** of a vector.	
6. Equal Vectors	If two vectors have the **same magnitude and direction**, they are **equal**.	
7. Parallel Vectors	**Parallel** vectors are **multiples** of each other.	2**a**+**b** and 4**a**+2**b** are parallel as they are multiple of each other. 

8. Collinear Vectors	**Collinear** vectors are vectors that are on the **same line**. To show that two vectors are **collinear**, show that one vector is a **multiple** of the other (parallel) **AND** that both vectors **share a point**.	
9. Resultant Vector	The **resultant** vector is the vector that results from **adding** two or more vectors together. The resultant can also be shown by **lining up** the **head** of one vector with the **tail** of the other.	if $\underline{a} = \begin{pmatrix}4\\4\end{pmatrix}$ and $\underline{b} = \begin{pmatrix}2\\-2\end{pmatrix}$ then $\underline{a} + \underline{b} = \begin{pmatrix}4\\4\end{pmatrix} + \begin{pmatrix}2\\-2\end{pmatrix} = \begin{pmatrix}6\\2\end{pmatrix}$ 
10. Scalar of a Vector	A **scalar** is the **number** we **multiply** a vector by.	 Example: 3a + 2b = $= 3\begin{pmatrix}2\\1\end{pmatrix} + 2\begin{pmatrix}4\\-1\end{pmatrix}$ $= \begin{pmatrix}6\\3\end{pmatrix} + \begin{pmatrix}8\\-2\end{pmatrix}$ $= \begin{pmatrix}14\\1\end{pmatrix}$
11. Vector Geometry	 $\overrightarrow{OA} = a \quad \overrightarrow{AO} = -a$ $\overrightarrow{OB} = b \quad \overrightarrow{BO} = -b$ $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -a + b = b - a$ $\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = -b + a = a - b$	**Example 1:** X is the midpoint of AB. Find $\overrightarrow{OX}$ **Answer:** Draw X on the original diagram  Now build up a journey. You could use $\overrightarrow{OX} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$. This will give: $\overrightarrow{OX} = a + \frac{1}{2}(b-a)$. This will simplify to $\frac{1}{2}a + \frac{1}{2}b$ or $\frac{1}{2}(a+b)$

Topic/Skill	Definition/Tips	Example
Circle Theorem 1	**Angles in a semi-circle have a right angle at the circumference.** 	 $y = 90°$ $x = 180 - 90 - 38 = 52°$
Circle Theorem 2	**Opposite angles in a cyclic quadrilateral add up to 180°.** 	 $x = 180 - 83 = 97°$ $y = 180 - 92 = 88°$
Circle Theorem 3	**The angle at the centre is twice the angle at the circumference.** 	 $x = 104 \div 2 = 52°$
Circle Theorem 4	**Angles in the same segment are equal.** 	 $x = 42°$ $y = 31°$
Circle Theorem 5	**A tangent is perpendicular to the radius at the point of contact.** 	 $y = 5cm$ (Pythagoras' Theorem)

Circle Theorem 6	**Tangents from an external point at equal in length.** 	 $x = 90°$
Circle Theorem 7	**Alternate Segment Theorem** 	 $x = 52°$ $y = 38°$

Higher Only Topics