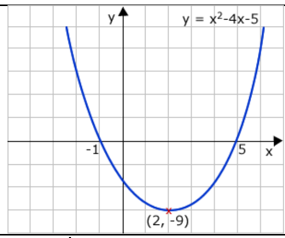
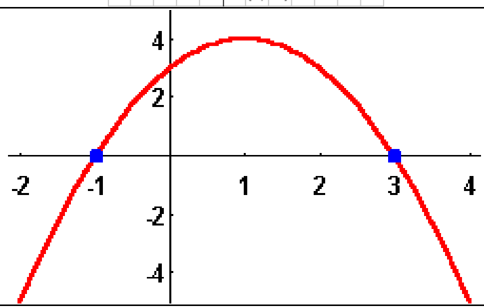
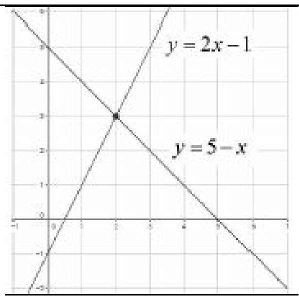
Topic: Quadratics

Topic/Skill	Definition/Tips	Example
1. Quadratic	A quadratic expression is of the form $\boldsymbol{ax^2 + bx + c}$ where a, b and c are numbers, $\boldsymbol{a \neq 0}$	Examples of quadratic expressions: x^2 $8x^2 - 3x + 7$ Examples of non-quadratic expressions: $2x^3 - 5x^2$ $9x - 1$
2. Factorising Quadratics	When a quadratic expression is in the form $x^2 + bx + c$ find the two numbers that **add to give b** and **multiply to give c**.	$x^2 + 7x + 10 = (x + 5)(x + 2)$ (because 5 and 2 add to give 7 and multiply to give 10) $x^2 + 2x - 8 = (x + 4)(x - 2)$ (because +4 and -2 add to give +2 and multiply to give -8)
3. Difference of Two Squares	An expression of the form $\boldsymbol{a^2 - b^2}$ can be factorised to give $(\boldsymbol{a} + \boldsymbol{b})(\boldsymbol{a} - \boldsymbol{b})$	$x^2 - 25 = (x + 5)(x - 5)$ $16x^2 - 81 = (4x + 9)(4x - 9)$
4. Solving Quadratics ($ax^2 = b$)	Isolate the x^2 term and square root both sides. Remember there will be a **positive and a negative solution**.	$2x^2 = 98$ $x^2 = 49$ $x = \pm 7$
5. Solving Quadratics ($ax^2 + bx = 0$)	**Factorise** and then **solve = 0**.	$x^2 - 3x = 0$ $x(x - 3) = 0$ $x = 0 \text{ or } x = 3$
6. Solving Quadratics by Factorising ($a = 1$)	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $x^2 + 3x - 10 = 0$ Factorise: $(x + 5)(x - 2) = 0$ $x = -5 \text{ or } x = 2$
7. Quadratic Graph	A '**U-shaped**' curve called a **parabola**. The equation is of the form $y = \boldsymbol{ax^2 + bx + c}$, where a, b and c are numbers, $\boldsymbol{a \neq 0}$. If $\boldsymbol{a < 0}$**,** the parabola is **upside down**.	
8. Roots of a Quadratic	A root is a **solution**. The roots of a quadratic are the ***x*-intercepts of the quadratic graph**.	

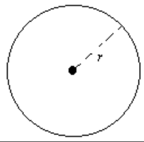
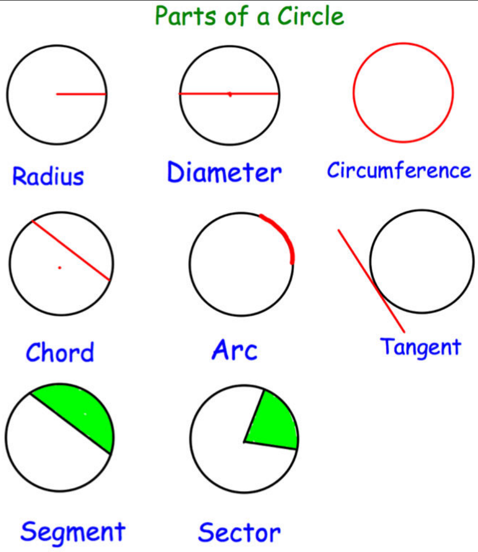
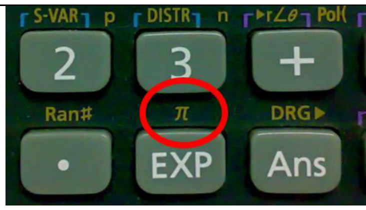
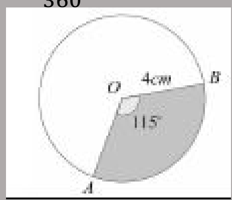
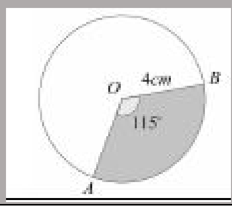
9. Turning Point of a Quadratic	A turning point is the **point where a quadratic turns**. On a **positive parabola**, the turning point is called a **minimum**. On a **negative parabola**, the turning point is called a **maximum**.	

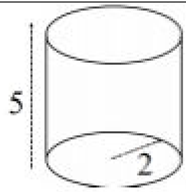
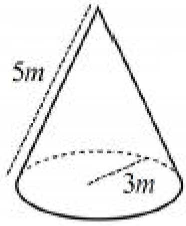
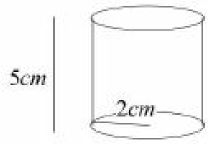
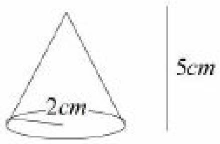
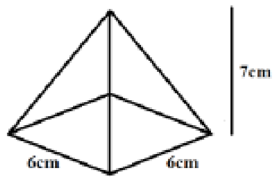
Topic/Skill	Definition/Tips	Example
1. Solve	To find the **answer**/value of something **Use inverse operations** on both sides of the equation (balancing method) until you find the value for the letter.	Solve $2x - 3 = 7$ Add 3 on both sides $2x = 10$ Divide by 2 on both sides $x = 5$
2. Inverse	**Opposite**	The inverse of addition is subtraction. The inverse of multiplication is division.
3. Rearranging Formulae	**Use inverse operations** on both sides of the formula (balancing method) until you find the expression for the letter.	Make x the subject of $y = \frac{2x-1}{z}$ Multiply both sides by z $yz = 2x - 1$ Add 1 to both sides $yz + 1 = 2x$ Divide by 2 on both sides $\frac{yz+1}{2} = x$ We now have x as the subject.
4. Writing Formulae	**Substitute letters for words** in the question.	Bob charges £3 per window and a £5 call out charge. $C = 3N + 5$ Where N=number of windows and C=cost
5. Substitution	**Replace letters with numbers**. Be careful of $5x^2$. You need to square first, then multiply by 5.	$a = 3, b = 2\ and\ c = 5$. Find: 1. $2a = 2 \times 3 = 6$ 2. $3a - 2b = 3 \times 3 - 2 \times 2 = 5$ 3. $7b^2 - 5 = 7 \times 2^2 - 5 = 23$

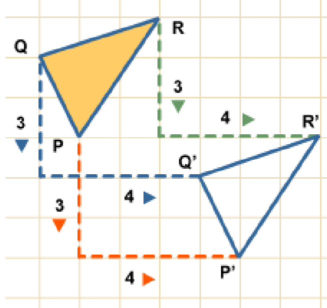
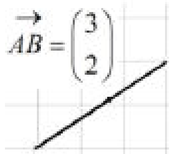
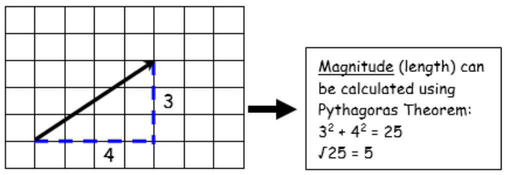
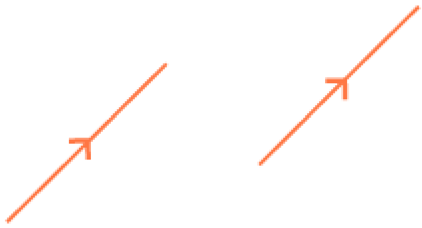
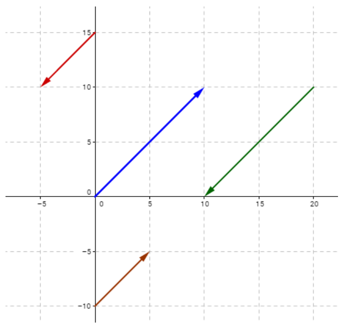
Topic/Skill	Definition/Tips	Example
1. Simultaneous Equations	A set of **two or more equations**, each involving **two or more variables** (letters). The **solutions** to simultaneous equations **satisfy both**/all of the **equations**.	$2x + y = 7$ $3x - y = 8$ $x = 3$ $y = 1$
2. Variable	A **symbol**, usually a **letter**, which **represents a number** which is usually unknown.	In the equation $x + 2 = 5$, x is the variable.
3. Coefficient	A **number** used to **multiply** a **variable**. It is the number that comes before/in front of a letter.	6z 6 is the coefficient z is the variable
4. Solving Simultaneous Equations (by Elimination)	1. **Balance** the **coefficients** of one of the variables. 2. **Eliminate** this variable by adding or subtracting the equations (**Same Sign Subtract, Different Sign Add**) 3. **Solve** the linear equation you get using the other variable. 4. **Substitute** the value you found back into one of the previous equations. 5. **Solve** the equation you get. 6. **Check** that the two values you get satisfy both of the original equations.	$5x + 2y = 9$ $10x + 3y = 16$ Multiply the first equation by 2. $10x + 4y = 18$ $10x + 3y = 16$ Same Sign Subtract (+10x on both) $y = 2$ Substitute $y = 2$ in to equation. $5x + 2 \times 2 = 9$ $5x + 4 = 9$ $5x = 5$ $x = 1$ Solution: $x = 1, y = 2$
5. Solving Simultaneous Equations (by Substitution)	1. **Rearrange** one of the equations into the form $y =\dots$ or $x =\dots$ 2. **Substitute** the right-hand side of the rearranged equation into the other equation. 3. Expand and **solve** this equation. 4. **Substitute** the value into the $y =\dots$ or $x =\dots$ equation. 5. **Check** that the two values you get satisfy both of the original equations.	$y - 2x = 3$ $3x + 4y = 1$ Rearrange: $y - 2x = 3 \rightarrow y = 2x + 3$ Substitute: $3x + 4(2x + 3) = 1$ Solve: $3x + 8x + 12 = 1$ $11x = -11$ $x = -1$ Substitute: $y = 2 \times -1 + 3$ $y = 1$ Solution: $x = -1, y = 1$

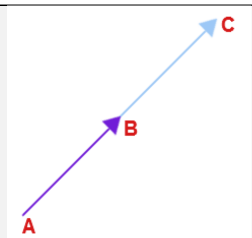
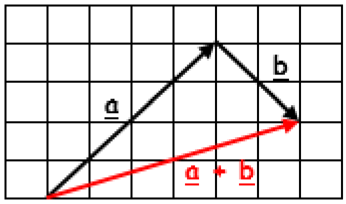
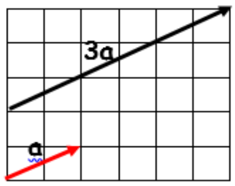
6. Solving Simultaneous Equations (Graphically)	**Draw the graphs** of the two equations. The **solutions** will be **where the lines meet**. The solution can be written as a **coordinate**.	 $y = 5 - x$ and $y = 2x - 1$. They meet at the point with coordinates (2,3) so the answer is $x = 2$ and $y = 3$

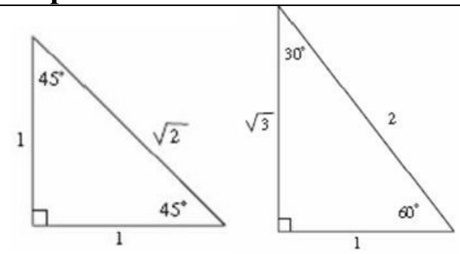
Topic: Circumference and Area

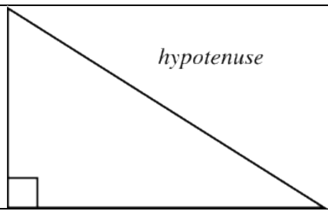
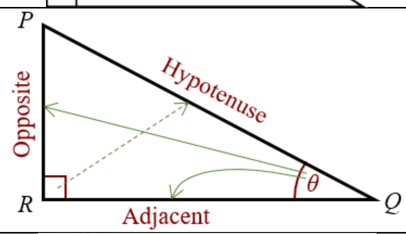
Topic/Skill	Definition/Tips	Example
1. Circle	A circle is the locus of all points equidistant from a central point.	
2. Parts of a Circle	**Radius** – the **distance** from the **centre** of a circle to the **edge** **Diameter** – the total **distance** across the **width** of a circle **through the centre**. **Circumference** – the **total distance** around the **outside** of a circle **Chord** – a **straight line** whose **end points lie on a circle** **Tangent** – a **straight line** which **touches** a circle at exactly **one point** **Arc** – a **part of the circumference** of a circle **Sector** – the **region** of a circle enclosed by **two radii** and their intercepted **arc** **Segment** – the **region** bounded by a **chord** and the **arc** created by the chord	
3. Area of a Circle	$\boldsymbol{A = \pi r^2}$ which means 'pi x radius squared'.	If the radius was 5cm, then: $A = \pi \times 5^2 = 78.5cm^2$
4. Circumference of a Circle	$\boldsymbol{C = \pi d}$ which means 'pi x diameter'	If the radius was 5cm, then: $C = \pi \times 10 = 31.4cm$
5. π ('pi')	Pi is the circumference of a circle divided by the diameter. $\boldsymbol{\pi \approx 3.14}$	
6. Arc Length of a Sector	The arc length is part of the circumference. Take the **angle** given **as a fraction over 360°** and **multiply** by the **circumference**.	Arc Length = $\frac{115}{360} \times \pi \times 8 = 8.03cm$ 
7. Area of a Sector	The area of a sector is part of the total area. Take the **angle** given **as a fraction over 360°** and **multiply** by the **area**.	Area = $\frac{115}{360} \times \pi \times 4^2 = 16.1cm^2$ 

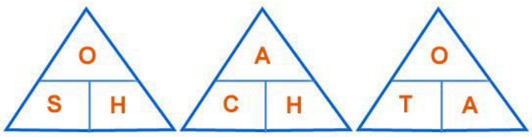
8. Surface Area of a Cylinder	**Curved Surface Area =** $\boldsymbol{\pi dh}$ or $\boldsymbol{2\pi rh}$ **Total SA =** $\boldsymbol{2\pi r^2 + \pi dh}$ or $\boldsymbol{2\pi r^2 + 2\pi rh}$	 $Total\ SA = 2\pi(2)^2 + \pi(4)(5) = 28\pi$
9. Surface Area of a Cone	**Curved Surface Area =** $\boldsymbol{\pi rl}$ where $l = slant\ height$ **Total SA =** $\boldsymbol{\pi rl + \pi r^2}$ You may need to use Pythagoras' Theorem to find the slant height	 $Total\ SA = \pi(3)(5) + \pi(3)^2 = 24\pi$
10. Surface Area of a Sphere	$\boldsymbol{SA = 4\pi r^2}$ Look out for hemispheres – halve the SA of a sphere and add on a circle (πr^2)	Find the surface area of a sphere with radius 3cm. $SA = 4\pi(3)^2 = 36\pi cm^2$
11. Volume of a Cylinder	$\boldsymbol{V = \pi r^2 h}$	 $V = \pi(4)(5)$ $= 62.8cm^3$
12. Volume of a Cone	$\boldsymbol{V = \frac{1}{3}\pi r^2 h}$	 $V = \frac{1}{3}\pi(4)(5)$ $= 20.9cm^3$
13. Volume of a Pyramid	$\boldsymbol{Volume = \frac{1}{3}Bh}$ where B = area of the base	 $V = \frac{1}{3} \times 6 \times 6 \times 7 = 84cm^3$
14. Volume of a Sphere	$\boldsymbol{V = \frac{4}{3}\pi r^3}$ Look out for hemispheres – just halve the volume of a sphere.	Find the volume of a sphere with diameter 10cm. $V = \frac{4}{3}\pi(5)^3 = \frac{500\pi}{3} cm^3$

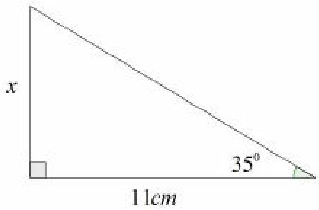
Topic/Skill	Definition/Tips	Example
1. Translation	**Translate** means to **move a shape**. The shape does not change **size** or **orientation**.	
2. Vector Notation	A vector can be written in 3 ways: **a** or $\overrightarrow{\boldsymbol{AB}}$ or $\begin{pmatrix} \mathbf{1} \\ \mathbf{3} \end{pmatrix}$	
3. Column Vector	In a column vector, the **top** number moves **left (-) or right (+)** and the **bottom** number moves **up (+) or down (-)**	$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ means '2 right, 3 up' $\begin{pmatrix} -1 \\ -5 \end{pmatrix}$ means '1 left, 5 down'
4. Vector	A **vector** is a quantity represented by an arrow with both **direction** and **magnitude**. $\overrightarrow{AB} = -\overrightarrow{BA}$	
5. Magnitude	Magnitude is defined as the **length** of a vector.	
6. Equal Vectors	If two vectors have the **same magnitude and direction**, they are **equal**.	
7. Parallel Vectors	**Parallel** vectors are **multiples** of each other.	2**a**+**b** and 4**a**+2**b** are parallel as they are multiple of each other. 

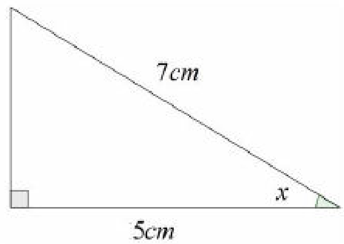
8. Collinear Vectors	**Collinear** vectors are vectors that are on the **same line**. To show that two vectors are **collinear**, show that one vector is a **multiple** of the other (parallel) **AND** that both vectors **share a point**.	
9. Resultant Vector	The **resultant** vector is the vector that results from **adding** two or more vectors together. The resultant can also be shown by **lining up** the **head** of one vector with the **tail** of the other.	if $\underline{a} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ then $\underline{a} + \underline{b} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ 
10. Scalar of a Vector	A **scalar** is the **number** we **multiply** a vector by.	 Example: 3a + 2b = $= 3\begin{pmatrix} 2 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ $= \begin{pmatrix} 6 \\ 3 \end{pmatrix} + \begin{pmatrix} 8 \\ -2 \end{pmatrix}$ $= \begin{pmatrix} 14 \\ 1 \end{pmatrix}$

<table>
<tr><th>Topic/Skill</th><th>Definition/Tips</th><th>Example</th></tr>
<tr><td>1. Exact Values for Angles in Trigonometry</td><td>
<table>
<tr><th></th><th>0°</th><th>30°</th><th>45°</th><th>60°</th><th>90°</th></tr>
<tr><td>sin</td><td>0</td><td>$\frac{1}{2}$</td><td>$\frac{\sqrt{2}}{2}$</td><td>$\frac{\sqrt{3}}{2}$</td><td>1</td></tr>
<tr><td>cos</td><td>1</td><td>$\frac{\sqrt{3}}{2}$</td><td>$\frac{\sqrt{2}}{2}$</td><td>$\frac{1}{2}$</td><td>0</td></tr>
<tr><td>tan</td><td>0</td><td>$\frac{1}{\sqrt{3}}$</td><td>1</td><td>$\sqrt{3}$</td><td>----</td></tr>
</table>
</td><td>
</td></tr>
<tr><td>2. Trigonometry</td><td>The study of triangles.</td><td></td></tr>
<tr><td>3. Hypotenuse</td><td>The longest side of a right-angled triangle.

Is always opposite the right angle.</td><td>
</td></tr>
<tr><td>4. Adjacent</td><td>Next to</td><td>
</td></tr>
<tr><td>5. Trigonometric Formulae</td><td>Use SOHCAHTOA.

$$\sin\theta = \frac{O}{H}$$
$$\cos\theta = \frac{A}{H}$$
$$\tan\theta = \frac{O}{A}$$


When finding a missing angle, use the ‘inverse’ trigonometric function by pressing the ‘shift’ button on the calculator.</td><td>

Use ‘Opposite’ and ‘Adjacent’, so use ‘tan’
$$\tan 35 = \frac{x}{11}$$
$$x = 11\tan 35 = 7.70cm$$


Use ‘Adjacent’ and ‘Hypotenuse’, so use ‘cos’
$$\cos x = \frac{5}{7}$$
$$x = cos^{-1}\left(\frac{5}{7}\right) = 44.4°$$</td></tr>
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