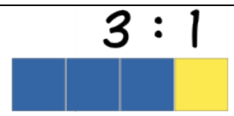
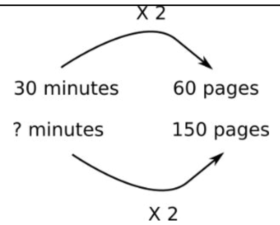
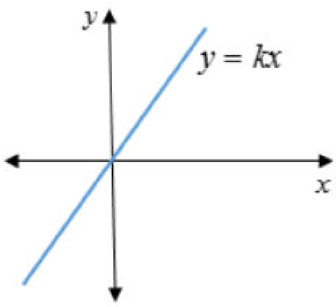
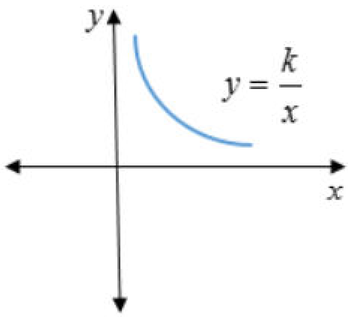
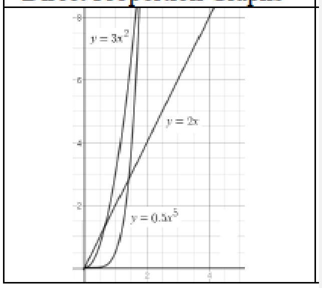
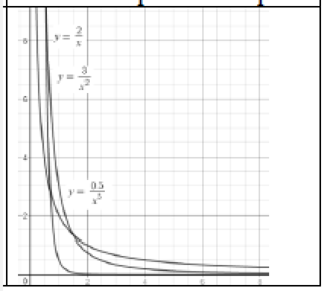
Topic/Skill	Definition/Tips	Example
1. Fraction	A mathematical expression representing the **division** of one integer by another. Fractions are written as **two numbers separated by a horizontal line**.	$\frac{2}{7}$ is a 'proper' fraction. $\frac{9}{4}$ is an 'improper' or 'top-heavy' fraction.
2. Numerator	The **top** number of a fraction.	In the fraction $\frac{3}{5}$, 3 is the numerator.
3. Denominator	The **bottom** number of a fraction.	In the fraction $\frac{3}{5}$, 5 is the denominator.
4. Unit Fraction	A fraction where the **numerator is one** and the denominator is a positive integer.	$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ $etc.$ are examples of unit fractions.
5. Reciprocal	The reciprocal of a number is **1 divided by the number**. The reciprocal of x is $\frac{1}{x}$ **When we multiply a number by its reciprocal we get 1**. This is called the 'multiplicative inverse'.	The reciprocal of 5 is $\frac{1}{5}$ The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, because $\frac{2}{3} \times \frac{3}{2} = 1$
6. Mixed Number	A number formed of both an **integer part** and a **fraction part**.	$3\frac{2}{5}$ is an example of a mixed number.
7. Simplifying Fractions	**Divide the numerator and denominator by the highest common factor**.	$\frac{20}{45} = \frac{4}{9}$
8. Equivalent Fractions	Fractions which represent the **same value**.	$\frac{2}{5} = \frac{4}{10} = \frac{20}{50} = \frac{60}{150}$ $etc.$
9. Comparing Fractions	To compare fractions, they each need to be rewritten so that they have a **common denominator**. **Ascending** means **smallest to biggest**. **Descending** means **biggest to smallest**.	Put in to ascending order : $\frac{3}{4}, \frac{2}{3}, \frac{5}{6}, \frac{1}{2}$. Equivalent: $\frac{9}{12}, \frac{8}{12}, \frac{10}{12}, \frac{6}{12}$ Correct order: $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$
10. Fraction of an Amount	**Divide** by the **bottom**, **times** by the **top**	Find $\frac{2}{5}$ of £60 $60 \div 5 = 12$ $12 \times 2 = 24$
11. Adding or Subtracting Fractions	Find the **LCM of the denominators** to find a common denominator. Use equivalent fractions to change each fraction to the **common denominator**.	$\frac{2}{3} + \frac{4}{5}$ Multiples of 3: 3, 6, 9, 12, **15**.. Multiples of 5: 5, 10, **15**.. LCM of 3 and 5 = 15

	Then just **add or subtract the numerators** and keep the **denominator the same**.	$\frac{2}{3}=\frac{10}{15}$ $\frac{4}{5}=\frac{12}{15}$ $\frac{10}{15}+\frac{12}{15}=\frac{22}{15}=1\frac{7}{15}$
12. Multiplying Fractions	**Multiply** the **numerators** together and **multiply** the **denominators** together.	$\frac{3}{8}\times\frac{2}{9}=\frac{6}{72}=\frac{1}{12}$
13. Dividing Fractions	**'Keep it, Flip it, Change it – KFC'** Keep the first fraction the same Flip the second fraction upside down Change the divide to a multiply Multiply by the reciprocal of the second fraction.	$\frac{3}{4}\div\frac{5}{6}=\frac{3}{4}\times\frac{6}{5}=\frac{18}{20}=\frac{9}{10}$

Topic/Skill	Definition/Tips	Example
1. Percentage	**Number of parts per 100.**	31% means $\frac{31}{100}$
2. Finding 10%	To find **10%**, **divide by 10**	10% of £36 = 36÷10=£3.60
3. Finding 1%	To find **1%**, **divide by 100**	1% of £8 = 8÷100 = £0.08
4. Percentage Change	$\frac{Difference}{Original} \times 100\%$	A games console is bought for £200 and sold for £250. % change = $\frac{50}{200} \times 100 = 25\%$
5. Fractions to Decimals	**Divide the numerator by the denominator** using the bus stop method.	$\frac{3}{8} = 3 \div 8 = 0.375$
6. Decimals to Fractions	**Write as a fraction** over 10, 100 or 1000 and simplify.	$0.36 = \frac{36}{100} = \frac{9}{25}$
7. Percentages to Decimals	**Divide by 100**	$8\% = 8 \div 100 = 0.08$
8. Decimals to Percentages	**Multiply by 100**	$0.4 = 0.4 \times 100\% = 40\%$
9. Fractions to Percentages	Percentage is just a fraction out of 100. **Make the denominator 100 using equivalent fractions**. When the denominator doesn't go in to 100, use a calculator and **multiply the fraction by 100**.	$\frac{3}{25} = \frac{12}{100} = 12\%$ $\frac{9}{17} \times 100 = 52.9\%$
10. Percentages to Fractions	Percentage is just a fraction out of 100. **Write the percentage over 100** and simplify.	$14\% = \frac{14}{100} = \frac{7}{50}$

Topic/Skill	Definition/Tips	Example
1. Increase or Decrease by a Percentage	Non-calculator: **Find the percentage** and **add** or **subtract** it from the **original** amount. Calculator: Find the **percentage multiplier** and multiply.	Increase 500 by 20% (Non Calc): 10% of 500 = 50 so 20% of 500 = 100 500 + 100 = 600 Decrease 800 by 17% (Calc): 100%-17%=83% 83% ÷ 100 = 0.83 0.83 x 800 = 664
2. Percentage Multiplier	The **number** you **multiply** a quantity by to **increase or decrease** it by a **percentage**.	The multiplier for increasing by 12% is 1.12 The multiplier for decreasing by 12% is 0.88 The multiplier for increasing by 100% is 2.
3. Reverse Percentage	Find the **correct percentage given in the question**, then work backwards to **find 100%** Look out for words like '**before**' or '**original**'	A jumper was priced at £48.60 after a 10% reduction. Find its original price. 100% - 10% = 90% 90% = £48.60 1% = £0.54 100% = £54
4. Simple Interest	Interest calculated as a **percentage of the original** amount.	£1000 invested for 3 years at 10% simple interest. 10% of £1000 = £100 Interest = $3 \times £100 = £300$

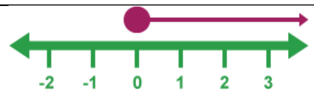
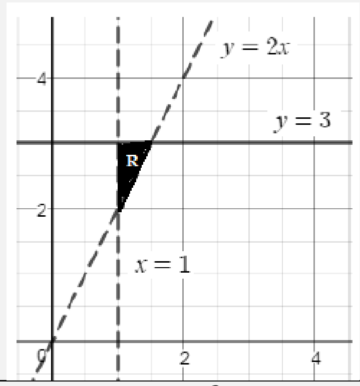
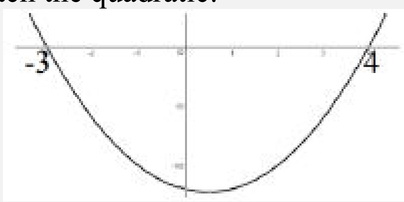
Topic/Skill	Definition/Tips	Example
1. Ratio	Ratio compares the size of **one part** to **another part**. Written using the ':' symbol.	
2. Proportion	Proportion compares the size of **one part** to the size of the **whole**. Usually written as a fraction.	In a class with 13 boys and 9 girls, the proportion of boys is $\frac{13}{22}$ and the proportion of girls is $\frac{9}{22}$
3. Simplifying Ratios	**Divide** all parts of the ratio by a **common factor**.	5 : 10 = 1 : 2 (divide both by 5) 14 : 21 = 2 : 3 (divide both by 7)
4. Ratios in the form $1 : n$ or $n : 1$	**Divide** both parts of the ratio by one of the numbers to make **one part equal 1**.	5 : 7 = 1 : $\frac{7}{5}$ in the form 1 : n 5 : 7 = $\frac{5}{7}$: 1 in the form n : 1
5. Sharing in a Ratio	**1. Add** the total parts of the ratio. **2. Divide** the amount to be shared by this value to find the value of one part. **3. Multiply** this value by each part of the ratio. Use only if you **know the total**.	Share £60 in the ratio 3 : 2 : 1. 3 + 2 + 1 = 6 60 ÷ 6 = 10 3 x 10 = 30, 2 x 10 = 20, 1 x 10 = 10 £30 : £20 : £10
6. Proportional Reasoning	Comparing two things using **multiplicative reasoning** and applying this to a new situation. Identify one multiplicative link and use this to find missing quantities.	
7. Unitary Method	Finding the **value of a single unit** and then finding the necessary value by **multiplying** the single unit value.	3 cakes require 450g of sugar to make. Find how much sugar is needed to make 5 cakes. 3 cakes = 450g So 1 cake = 150g (÷ by 3) So 5 cakes = 750 g (x by 5)
8. Ratio already shared	Find what **one part** of the ratio is worth using the **unitary method**.	Money was shared in the ratio 3:2:5 between Ann, Bob and Cat. Given that Bob had £16, found out the total amount of money shared. £16 = 2 parts So £8 = 1 part 3 + 2 + 5 = 10 parts, so 8 x 10 = £80
9. Best Buys	Find the **unit cost** by **dividing** the **price by the quantity**. The **lowest** number is the best value.	8 cakes for £1.28 → 16p each (÷by 8) 13 cakes for £2.05 → 15.8p each (÷by 13) Pack of 13 cakes is best value.

Topic/Skill	Definition/Tips	Example
1. Direct Proportion	If two quantities are in direct proportion, **as one increases**, the **other increases** by the **same percentage**. If y is directly proportional to x, this can be written as $\boldsymbol{y \propto x}$ An equation of the form $\boldsymbol{y = kx}$ represents direct proportion, where k **is the constant of proportionality**.	
2. Inverse Proportion	If two quantities are inversely proportional, **as one increases**, the **other decreases** by the **same percentage**. If y is inversely proportional to x, this can be written as $\boldsymbol{y \propto \frac{1}{x}}$ An equation of the form $\boldsymbol{y = \frac{k}{x}}$ represents inverse proportion.	
3. Using proportionality formulae	**Direct**: **y = kx** or **y ∝ x** **Inverse**: **y** = $\frac{k}{x}$ or **y** ∝ $\frac{1}{x}$ 1. **Solve to find k** using the pair of values in the question. 2. **Rewrite the equation** using the k you have just found. 3. **Substitute the other given value** from the question in to the equation to **find the missing value**.	p is directly proportional to q. When p = 12, q = 4. Find p when q = 20. 1. p = kq 12 = k x 4 so k = 3 2. p = 3q 3. p = 3 x 20 = 60, so p = 60
4. Direct Proportion with powers	Graphs showing **direct proportion** can be written in the form $\boldsymbol{y = kx^n}$ Direct proportion graphs will always start at the origin.	Direct Proportion Graphs 
5. Inverse Proportion with powers	Graphs showing **inverse proportion** can be written in the form $\boldsymbol{y = \frac{k}{x^n}}$ Inverse proportion graphs will never start at the origin.	Inverse Proportion Graphs 

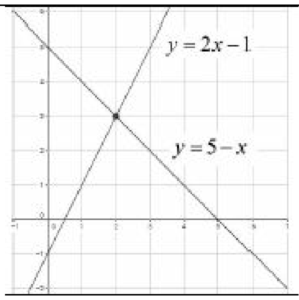
Topic: Solving Quadratics by Factorising

Topic/Skill	Definition/Tips	Example
1. Quadratic	A quadratic expression is of the form $\boldsymbol{ax^2 + bx + c}$ where a, b and c are numbers, $\boldsymbol{a \neq 0}$	Examples of quadratic expressions: x^2 $8x^2 - 3x + 7$ Examples of non-quadratic expressions: $2x^3 - 5x^2$ $9x - 1$
2. Factorising Quadratics	When a quadratic expression is in the form $x^2 + bx + c$ find the two numbers that **add to give b** and **multiply to give c**.	$x^2 + 7x + 10 = (x + 5)(x + 2)$ (because 5 and 2 add to give 7 and multiply to give 10) $x^2 + 2x - 8 = (x + 4)(x - 2)$ (because +4 and -2 add to give +2 and multiply to give -8)
3. Difference of Two Squares	An expression of the form $\boldsymbol{a^2 - b^2}$ can be factorised to give $(\boldsymbol{a} + \boldsymbol{b})(\boldsymbol{a} - \boldsymbol{b})$	$x^2 - 25 = (x + 5)(x - 5)$ $16x^2 - 81 = (4x + 9)(4x - 9)$
4. Solving Quadratics ($ax^2 = b$)	Isolate the x^2 term and square root both sides. Remember there will be a **positive and a negative solution**.	$2x^2 = 98$ $x^2 = 49$ $x = \pm 7$
5. Solving Quadratics ($ax^2 + bx = 0$)	**Factorise** and then **solve = 0**.	$x^2 - 3x = 0$ $x(x - 3) = 0$ $x = 0 \; or \; x = 3$
6. Solving Quadratics by Factorising ($a = 1$)	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $x^2 + 3x - 10 = 0$ Factorise: $(x + 5)(x - 2) = 0$ $x = -5 \; or \; x = 2$
7. Factorising Quadratics when $a \neq 1$	When a quadratic is in the form $ax^2 + bx + c$ 1. Multiply a by c = ac 2. Find two numbers that add to give b and multiply to give ac. 3. Re-write the quadratic, replacing bx with the two numbers you found. 4. Factorise in pairs – you should get the same bracket twice 5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets.	Factorise $6x^2 + 5x - 4$ 1. $6 \times -4 = -24$ 2. Two numbers that add to give +5 and multiply to give -24 are +8 and -3 3. $6x^2 + 8x - 3x - 4$ 4. Factorise in pairs: $2x(3x + 4) - 1(3x + 4)$ 5. Answer = $(3x + 4)(2x - 1)$
8. Solving Quadratics by Factorising ($a \neq 1$)	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $2x^2 + 7x - 4 = 0$ Factorise: $(2x - 1)(x + 4) = 0$ $x = \frac{1}{2} \; or \; x = -4$

9. Completing the Square (when $a = 1$)	A quadratic in the form $x^2 + bx + c$ can be written in the form $(\boldsymbol{x} + \boldsymbol{p})^2 + \boldsymbol{q}$ 1. Write a set of brackets with x in and **half** the value of b. 2. Square the bracket. 3. Subtract $\left(\frac{b}{2}\right)^2$ and add c. 4. Simplify the expression. You can **use the completing the square form** to help **find the maximum or minimum** of quadratic graph.	Complete the square of $y = x^2 - 6x + 2$ Answer: $(x-3)^2 - 3^2 + 2$ $= (x-3)^2 - 7$ The minimum value of this expression occurs when $(x-3)^2 = 0$, which occurs when $x = 3$ When $x = 3$, $y = 0 - 7 = -7$ Minimum point = $(3, -7)$
10. Completing the Square (when $a \neq 1$)	A quadratic in the form $ax^2 + bx + c$ can be written in the form $\mathbf{p}(\boldsymbol{x} + \boldsymbol{q})^2 + \boldsymbol{r}$ Use the same method as above, but factorise out a at the start.	Complete the square of $4x^2 + 8x - 3$ Answer: $4[x^2 + 2x] - 3$ $= 4[(x+1)^2 - 1^2] - 3$ $= 4(x+1)^2 - 4 - 3$ $= 4(x+1)^2 - 7$
11. Solving Quadratics by Completing the Square	**Complete the square** in the usual way and **use inverse operations to solve**.	Solve $x^2 + 8x + 1 = 0$ Answer: $(x+4)^2 - 4^2 + 1 = 0$ $(x+4)^2 - 15 = 0$ $(x+4)^2 = 15$ $(x+4) = \pm\sqrt{15}$ $x = -4 \pm \sqrt{15}$
12. Solving Quadratics using the Quadratic Formula	A quadratic in the form $ax^2 + bx + c = 0$ can be solved using the formula: $\boldsymbol{x} = \frac{-\boldsymbol{b} \pm \sqrt{\boldsymbol{b}^2 - 4\boldsymbol{ac}}}{2\boldsymbol{a}}$ Use the formula if the quadratic does not factorise easily.	Solve $3x^2 + x - 5 = 0$ Answer: $a = 3, b = 1, c = -5$ $x = \frac{-1 \pm \sqrt{1^2 - 4 \times 3 \times -5}}{2 \times 3}$ $x = \frac{-1 \pm \sqrt{61}}{6}$ $x = 1.14\ or - 1.47\ (2\ d.p.)$

Topic/Skill	Definition/Tips	Example
1. Inequality	An inequality says that two values are **not equal**. $a \neq b$ means that a is not equal to b.	$7 \neq 3$ $x \neq 0$
2. Inequality symbols	$x > 2$ **means x is greater than 2** $x < 3$ **means x is less than 3** $x \geq 1$ **means x is greater than or equal to 1** $x \leq 6$ **means x is less than or equal to 6**	State the integers that satisfy $-2 < x \leq 4$. -1, 0, 1, 2, 3, 4
3. Inequalities on a Number Line	Inequalities can be shown on a number line. **Open circles** are used for numbers that are **less than or greater than** $(< or >)$ **Closed circles** are used for numbers that are **less than or equal or greater than or equal** $(\leq or \geq)$	 $x \geq 0$  $x < 2$  $-5 \leq x < 4$
4. Graphical Inequalities	Inequalities can be represented on a coordinate grid. If the inequality is **strict** $(x > 2)$ then use a **dotted line**. If the inequality is **not strict** $(x \leq 6)$ then use a **solid line**. **Shade** the **region** which satisfies all the inequalities.	Shade the region that satisfies: $y > 2x, x > 1 \; and \; y \leq 3$ 
5. Quadratic Inequalities	**Sketch the quadratic graph** of the inequality. If the expression is $>$ ***or*** $\geq$ then the answer will be **above the x-axis**. If the expression is $<$ ***or*** $\leq$ then the answer will be **below the x-axis**. Look carefully at the inequality symbol in the question. Look carefully if the quadratic is a **positive or negative parabola**.	Solve the inequality $x^2 - x - 12 < 0$ Sketch the quadratic:  The required region is below the x-axis, so the final answer is: $-3 < x < 4$ If the question had been > 0, the answer would have been: $x < -3 \; or \; x > 4$

Topic/Skill	Definition/Tips	Example
1. Simultaneous Equations	A set of **two or more equations**, each involving **two or more variables** (letters). The **solutions** to simultaneous equations **satisfy both**/all of the **equations**.	$2x + y = 7$ $3x - y = 8$ $x = 3$ $y = 1$
2. Variable	A **symbol**, usually a **letter**, which **represents a number** which is usually unknown.	In the equation $x + 2 = 5$, x is the variable.
3. Coefficient	A **number** used to **multiply** a **variable**. It is the number that comes before/in front of a letter.	6z 6 is the coefficient z is the variable
4. Solving Simultaneous Equations (by Elimination)	1. **Balance** the **coefficients** of one of the variables. 2. **Eliminate** this variable by adding or subtracting the equations (**Same Sign Subtract, Different Sign Add**) 3. **Solve** the linear equation you get using the other variable. 4. **Substitute** the value you found back into one of the previous equations. 5. **Solve** the equation you get. 6. **Check** that the two values you get satisfy both of the original equations.	$5x + 2y = 9$ $10x + 3y = 16$ Multiply the first equation by 2. $10x + 4y = 18$ $10x + 3y = 16$ Same Sign Subtract (+10x on both) $y = 2$ Substitute $y = 2$ in to equation. $5x + 2 \times 2 = 9$ $5x + 4 = 9$ $5x = 5$ $x = 1$ Solution: $x = 1, y = 2$
5. Solving Simultaneous Equations (by Substitution)	1. **Rearrange** one of the equations into the form $y = ...$ or $x = ...$ 2. **Substitute** the right-hand side of the rearranged equation into the other equation. 3. Expand and **solve** this equation. 4. **Substitute** the value into the $y = ...$ or $x = ...$ equation. 5. **Check** that the two values you get satisfy both of the original equations.	$y - 2x = 3$ $3x + 4y = 1$ Rearrange: $y - 2x = 3 \rightarrow y = 2x + 3$ Substitute: $3x + 4(2x + 3) = 1$ Solve: $3x + 8x + 12 = 1$ $11x = -11$ $x = -1$ Substitute: $y = 2 \times -1 + 3$ $y = 1$ Solution: $x = -1, y = 1$

6. Solving Simultaneous Equations (Graphically)	**Draw the graphs** of the two equations. The **solutions** will be **where the lines meet**. The solution can be written as a **coordinate**.	 $y = 5 - x$ and $y = 2x - 1$. They meet at the point with coordinates (2,3) so the answer is $x = 2$ and $y = 3$
7. Solving Linear and Quadratic Simultaneous Equations	Method 1: If both equations are in the same form (eg. Both $y =\ldots$): 1. Set the equations **equal to each other**. 2. **Rearrange** to make the equation **equal to zero**. 3. **Solve** the quadratic equation. 4. **Substitute** the values back in to one of the equations. Method 2: If the equations are not in the same form: 1. **Rearrange** the linear equation into the form $y =\ldots$ or $x =\ldots$ 2. **Substitute** in to the quadratic equation. 3. **Rearrange** to make the equation **equal to zero**. 4. **Solve** the quadratic equation. 5. **Substitute** the values back in to one of the equations. You should get **two pairs of solutions** (two values for x, two values for y.) Graphically, you should have **two points of intersection**.	<u>Example 1</u> Solve $y = x^2 - 2x - 5$ and $y = x - 1$ $x^2 - 2x - 5 = x - 1$ $x^2 - 3x - 4 = 0$ $(x - 4)(x + 1) = 0$ $x = 4$ and $x = -1$ $y = 4 - 1 = 3$ and $y = -1 - 1 = -2$ Answers: (4,3) and (-1,-2) <u>Example 2</u> Solve $x^2 + y^2 = 5$ and $x + y = 3$ $x = 3 - y$ $(3 - y)^2 + y^2 = 5$ $9 - 6y + y^2 + y^2 = 5$ $2y^2 - 6y + 4 = 0$ $y^2 - 3y + 2 = 0$ $(y - 1)(y - 2) = 0$ $y = 1$ and $y = 2$ $x = 3 - 1 = 2$ and $x = 3 - 2 = 1$ Answers: (2,1) and (1,2)