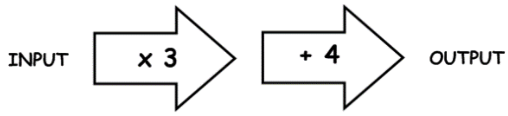
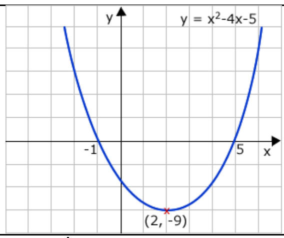
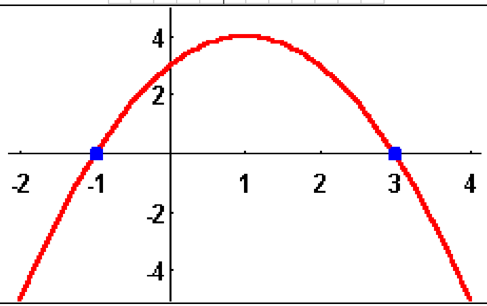
Topic/Skill	Definition/Tips	Example
1. Algebraic Fraction	A fraction whose **numerator** and **denominator** are **algebraic expressions**.	$\frac{6x}{3x-1}$
2. Adding/ Subtracting Algebraic Fractions	For $\frac{a}{b} \pm \frac{c}{d}$, the **common denominator** is bd $\frac{a}{b} \pm \frac{c}{d} = \frac{ad}{bd} \pm \frac{bc}{bd} = \frac{ad \pm bc}{bd}$	$\frac{1}{x} + \frac{x}{2y}$ $= \frac{1(2y)}{2xy} + \frac{x(x)}{2xy}$ $= \frac{2y + x^2}{2xy}$
3. Multiplying Algebraic Fractions	**Multiply** the **numerators together** and the **denominators together**. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$	$\frac{x}{3} \times \frac{x+2}{x-2}$ $= \frac{x(x+2)}{3(x-2)}$ $= \frac{x^2+2x}{3x-6}$
4. Dividing Algebraic Fractions	**Multiply** the first fraction by the **reciprocal of the second fraction**. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$	$\frac{x}{3} \div \frac{2x}{7}$ $= \frac{x}{3} \times \frac{7}{2x}$ $= \frac{7x}{6x} = \frac{7}{6}$
5. Simplifying Algebraic Fractions	**Factorise** the numerator and denominator and **cancel common factors**.	$\frac{x^2+x-6}{2x-4} = \frac{(x+3)(x-2)}{2(x-2)} = \frac{x+3}{2}$

Topic/Skill	Definition/Tips	Example
1. Function Machine	Takes an **input** value, performs some **operations** and produces an **output** value.	
2. Function	A **relationship** between two sets of values.	$f(x) = 3x^2 - 5$ 'For any input value, square the term, then multiply by 3, then subtract 5'.
3. Function notation	$f(x)$ $\boldsymbol{x}$ is the **input** value $\boldsymbol{f(x)}$ is the **output** value.	$f(x) = 3x + 11$ Suppose the input value is $x = 5$ The output value is $f(5) = 3 \times 5 + 11 = 26$
4. Inverse function	$f^{-1}(x)$ A function that performs the **opposite process** of the original function. 1. Write the function as $y = f(x)$ 2. Rearrange to make x the subject. 3. Replace the $\boldsymbol{y}$ **with** $\boldsymbol{x}$ and the $\boldsymbol{x}$ **with** $\boldsymbol{f^{-1}(x)}$	$f(x) = (1-2x)^5$. Find the inverse. $y = (1-2x)^5$ $\sqrt[5]{y} = 1 - 2x$ $1 - \sqrt[5]{y} = 2x$ $\frac{1-\sqrt[5]{y}}{2} = x$ $f^{-1}(x) = \dfrac{1 - \sqrt[5]{x}}{2}$
5. Composite function	A **combination** of two or more **functions** to create a new function. $\boldsymbol{fg(x)}$ is the composite function that **substitutes** the function $\boldsymbol{g(x)}$ **into** the function $\boldsymbol{f(x)}$. $\boldsymbol{fg(x)}$ means '**do g first, then f**' $\boldsymbol{gf(x)}$ means '**do f first, then g**'	$f(x) = 5x - 3$, $g(x) = \frac{1}{2}x + 1$ What is $fg(4)$? $g(4) = \dfrac{1}{2} \times 4 + 1 = 3$ $f(3) = 5 \times 3 - 3 = 12 = fg(4)$ What is $fg(x)$? $fg(x) = 5\left(\dfrac{1}{2}x + 1\right) - 3 = \dfrac{5}{2}x + 2$

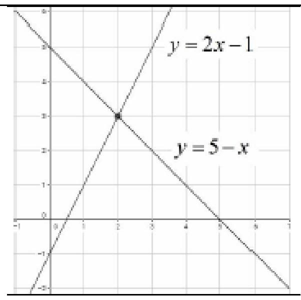
Topic: Further Quadratics

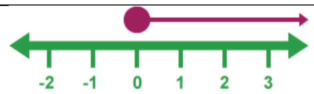
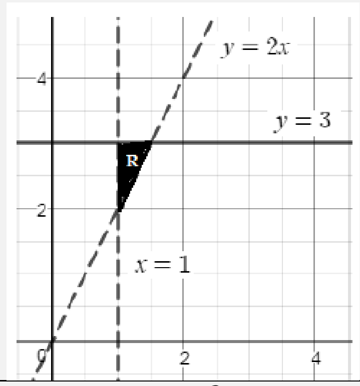
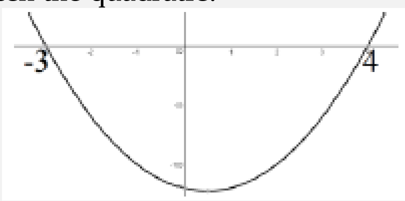
Topic/Skill	Definition/Tips	Example
1. Quadratic	A quadratic expression is of the form $$\boldsymbol{ax^2 + bx + c}$$ where a, b and c are numbers, $\boldsymbol{a \neq 0}$	Examples of quadratic expressions: x^2 $8x^2 - 3x + 7$ Examples of non-quadratic expressions: $2x^3 - 5x^2$ $9x - 1$
2. Factorising Quadratics	When a quadratic expression is in the form $x^2 + bx + c$ find the two numbers that **add to give b** and **multiply to give c**.	$x^2 + 7x + 10 = (x + 5)(x + 2)$ (because 5 and 2 add to give 7 and multiply to give 10) $x^2 + 2x - 8 = (x + 4)(x - 2)$ (because +4 and -2 add to give +2 and multiply to give -8)
3. Difference of Two Squares	An expression of the form $\boldsymbol{a^2 - b^2}$ can be factorised to give $(\boldsymbol{a + b})(\boldsymbol{a - b})$	$x^2 - 25 = (x + 5)(x - 5)$ $16x^2 - 81 = (4x + 9)(4x - 9)$
4. Solving Quadratics ($ax^2 = b$)	Isolate the x^2 term and square root both sides. Remember there will be a **positive and a negative solution**.	$2x^2 = 98$ $x^2 = 49$ $x = \pm 7$
5. Solving Quadratics ($ax^2 + bx = 0$)	**Factorise** and then **solve = 0**.	$x^2 - 3x = 0$ $x(x - 3) = 0$ $x = 0 \; or \; x = 3$
6. Solving Quadratics by Factorising ($a = 1$)	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $x^2 + 3x - 10 = 0$ Factorise: $(x + 5)(x - 2) = 0$ $x = -5 \; or \; x = 2$
7. Quadratic Graph	A '**U-shaped**' curve called a **parabola**. The equation is of the form $y = \boldsymbol{ax^2 + bx + c}$, where a, b and c are numbers, $\boldsymbol{a \neq 0}$. If $\boldsymbol{a < 0,}$ the parabola is **upside down**.	
8. Roots of a Quadratic	A root is a **solution**. The roots of a quadratic are the ***x*-intercepts of the quadratic graph**.	

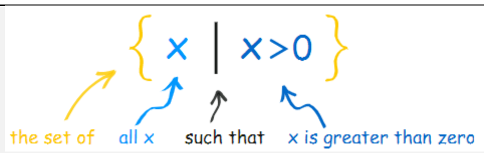
9. Turning Point of a Quadratic	A turning point is the **point where a quadratic turns**. On a **positive parabola**, the turning point is called a **minimum**. On a **negative parabola**, the turning point is called a **maximum**.	
10. Factorising Quadratics when $a \neq 1$	When a quadratic is in the form $ax^2 + bx + c$ 1. Multiply a by c = ac 2. Find two numbers that add to give b and multiply to give ac. 3. Re-write the quadratic, replacing bx with the two numbers you found. 4. Factorise in pairs – you should get the same bracket twice 5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets.	Factorise $6x^2 + 5x - 4$ 1. $6 \times -4 = -24$ 2. Two numbers that add to give +5 and multiply to give -24 are +8 and -3 3. $6x^2 + 8x - 3x - 4$ 4. Factorise in pairs: $2x(3x + 4) - 1(3x + 4)$ 5. Answer = $(3x + 4)(2x - 1)$
11. Solving Quadratics by Factorising $(a \neq 1)$	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $2x^2 + 7x - 4 = 0$ Factorise: $(2x - 1)(x + 4) = 0$ $x = \frac{1}{2}\ or\ x = -4$
12. Completing the Square (when $a = 1$)	A quadratic in the form $x^2 + bx + c$ can be written in the form $(\boldsymbol{x} + \boldsymbol{p})^2 + \boldsymbol{q}$ 1. Write a set of brackets with x in and **half** the value of b. 2. Square the bracket. 3. Subtract $\left(\frac{b}{2}\right)^2$ and add c. 4. Simplify the expression. You can **use the completing the square form** to help **find the maximum or minimum** of quadratic graph.	Complete the square of $y = x^2 - 6x + 2$ Answer: $(x - 3)^2 - 3^2 + 2$ $= (x - 3)^2 - 7$ The minimum value of this expression occurs when $(x - 3)^2 = 0$, which occurs when $x = 3$ When $x = 3$, $y = 0 - 7 = -7$ Minimum point = $(3, -7)$
13. Completing the Square (when $a \neq 1$)	A quadratic in the form $ax^2 + bx + c$ can be written in the form $\mathbf{p}(\boldsymbol{x} + \boldsymbol{q})^2 + \boldsymbol{r}$ Use the same method as above, but factorise out a at the start.	Complete the square of $4x^2 + 8x - 3$ Answer: $4[x^2 + 2x] - 3$ $= 4[(x + 1)^2 - 1^2] - 3$ $= 4(x + 1)^2 - 4 - 3$ $= 4(x + 1)^2 - 7$
14. Solving Quadratics by Completing the Square	**Complete the square** in the usual way and **use inverse operations to solve**.	Solve $x^2 + 8x + 1 = 0$ Answer: $(x + 4)^2 - 4^2 + 1 = 0$ $(x + 4)^2 - 15 = 0$

		$(x+4)^2 = 15$ $(x+4) = \pm\sqrt{15}$ $x = -4 \pm \sqrt{15}$
15. Solving Quadratics using the Quadratic Formula	A quadratic in the form $ax^2 + bx + c = 0$ can be solved using the formula: $\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ Use the formula if the quadratic does not factorise easily.	Solve $3x^2 + x - 5 = 0$ Answer: $a = 3, b = 1, c = -5$ $x = \frac{-1 \pm \sqrt{1^2 - 4 \times 3 \times -5}}{2 \times 3}$ $x = \frac{-1 \pm \sqrt{61}}{6}$ $x = 1.14\ or - 1.47\ (2\ d.p.)$

Topic/Skill	Definition/Tips	Example
1. Simultaneous Equations	A set of **two or more equations**, each involving **two or more variables** (letters). The **solutions** to simultaneous equations **satisfy both**/all of the **equations**.	$2x + y = 7$ $3x - y = 8$ $x = 3$ $y = 1$
2. Variable	A **symbol**, usually a **letter**, which **represents a number** which is usually unknown.	In the equation $x + 2 = 5$, x is the variable.
3. Coefficient	A **number** used to **multiply** a **variable**. It is the number that comes before/in front of a letter.	6z 6 is the coefficient z is the variable
4. Solving Simultaneous Equations (by Elimination)	1. **Balance** the **coefficients** of one of the variables. 2. **Eliminate** this variable by adding or subtracting the equations (**Same Sign Subtract, Different Sign Add**) 3. **Solve** the linear equation you get using the other variable. 4. **Substitute** the value you found back into one of the previous equations. 5. **Solve** the equation you get. 6. **Check** that the two values you get satisfy both of the original equations.	$5x + 2y = 9$ $10x + 3y = 16$ Multiply the first equation by 2. $10x + 4y = 18$ $10x + 3y = 16$ Same Sign Subtract (+10x on both) $y = 2$ Substitute $y = 2$ in to equation. $5x + 2 \times 2 = 9$ $5x + 4 = 9$ $5x = 5$ $x = 1$ Solution: $x = 1, y = 2$
5. Solving Simultaneous Equations (by Substitution)	1. **Rearrange** one of the equations into the form $y = ...$ or $x = ...$ 2. **Substitute** the right-hand side of the rearranged equation into the other equation. 3. Expand and **solve** this equation. 4. **Substitute** the value into the $y = ...$ or $x = ...$ equation. 5. **Check** that the two values you get satisfy both of the original equations.	$y - 2x = 3$ $3x + 4y = 1$ Rearrange: $y - 2x = 3 \rightarrow y = 2x + 3$ Substitute: $3x + 4(2x + 3) = 1$ Solve: $3x + 8x + 12 = 1$ $11x = -11$ $x = -1$ Substitute: $y = 2 \times -1 + 3$ $y = 1$ Solution: $x = -1, y = 1$

6. Solving Simultaneous Equations (Graphically)	**Draw the graphs** of the two equations. The **solutions** will be **where the lines meet**. The solution can be written as a **coordinate**.	 $y = 5 - x$ and $y = 2x - 1$. They meet at the point with coordinates (2,3) so the answer is $x = 2$ and $y = 3$
7. Solving Linear and Quadratic Simultaneous Equations	Method 1: If both equations are in the same form (eg. Both $y = \ldots$): 1. Set the equations **equal to each other**. 2. **Rearrange** to make the equation **equal to zero**. 3. **Solve** the quadratic equation. 4. **Substitute** the values back in to one of the equations. Method 2: If the equations are not in the same form: 1. **Rearrange** the linear equation into the form $y = \ldots$ or $x = \ldots$ 2. **Substitute** in to the quadratic equation. 3. **Rearrange** to make the equation **equal to zero**. 4. **Solve** the quadratic equation. 5. **Substitute** the values back in to one of the equations. You should get **two pairs of solutions** (two values for x, two values for y.) Graphically, you should have **two points of intersection**.	<u>Example 1</u> Solve $y = x^2 - 2x - 5$ and $y = x - 1$ $x^2 - 2x - 5 = x - 1$ $x^2 - 3x - 4 = 0$ $(x - 4)(x + 1) = 0$ $x = 4$ and $x = -1$ $y = 4 - 1 = 3$ and $y = -1 - 1 = -2$ Answers: (4,3) and (-1,-2) <u>Example 2</u> Solve $x^2 + y^2 = 5$ and $x + y = 3$ $x = 3 - y$ $(3 - y)^2 + y^2 = 5$ $9 - 6y + y^2 + y^2 = 5$ $2y^2 - 6y + 4 = 0$ $y^2 - 3y + 2 = 0$ $(y - 1)(y - 2) = 0$ $y = 1$ and $y = 2$ $x = 3 - 1 = 2$ and $x = 3 - 2 = 1$ Answers: (2,1) and (1,2)

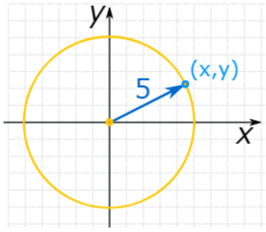
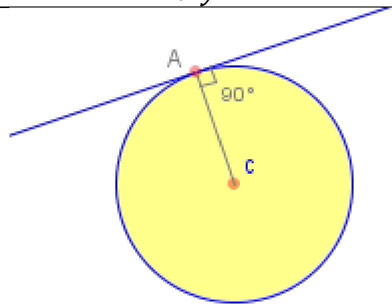
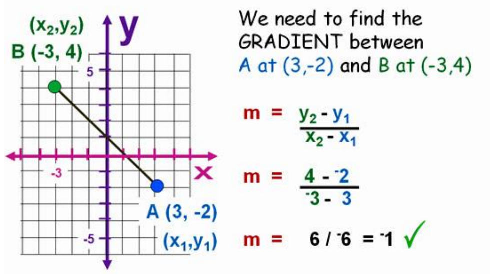
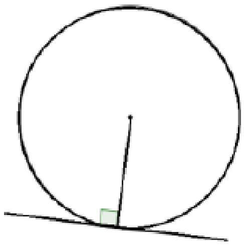
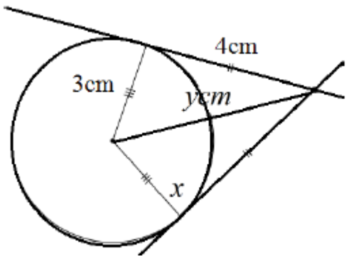
Topic/Skill	**Definition/Tips**	**Example**
1. Inequality	An inequality says that two values are **not equal**. $a \neq b$ means that a is not equal to b.	$7 \neq 3$ $x \neq 0$
2. Inequality symbols	$\boldsymbol{x > 2}$ means **x is greater than 2** $\boldsymbol{x < 3}$ means **x is less than 3** $\boldsymbol{x \geq 1}$ means **x is greater than or equal to 1** $\boldsymbol{x \leq 6}$ means **x is less than or equal to 6**	State the integers that satisfy $-2 < x \leq 4$. -1, 0, 1, 2, 3, 4
3. Inequalities on a Number Line	Inequalities can be shown on a number line. **Open circles** are used for numbers that are **less than or greater than** $(< or >)$ **Closed circles** are used for numbers that are **less than or equal or greater than or equal** $(\leq or \geq)$	 $x \geq 0$  $x < 2$  $-5 \leq x < 4$
4. Graphical Inequalities	Inequalities can be represented on a coordinate grid. If the inequality is **strict** $(x > 2)$ then use a **dotted line**. If the inequality is **not strict** $(x \leq 6)$ then use a **solid line**. **Shade** the **region** which satisfies all the inequalities.	Shade the region that satisfies: $y > 2x, x > 1$ *and* $y \leq 3$ 
5. Quadratic Inequalities	**Sketch the quadratic graph** of the inequality. If the expression is $>$ ***or*** $\geq$ then the answer will be **above the x-axis**. If the expression is $<$ ***or*** $\leq$ then the answer will be **below the x-axis**. Look carefully at the inequality symbol in the question. Look carefully if the quadratic is a **positive or negative parabola**.	Solve the inequality $x^2 - x - 12 < 0$ Sketch the quadratic:  The required region is below the x-axis, so the final answer is: $-3 < x < 4$ If the question had been > 0, the answer would have been: $x < -3$ *or* $x > 4$
6. Set Notation	A **set** is a **collection of things**, usually numbers, denoted with brackets { }	$\{3, 6, 9\}$ is a set.

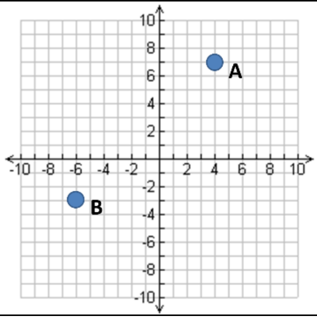
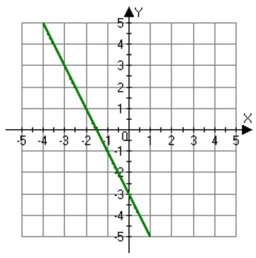
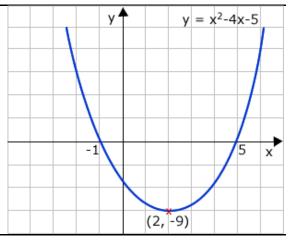
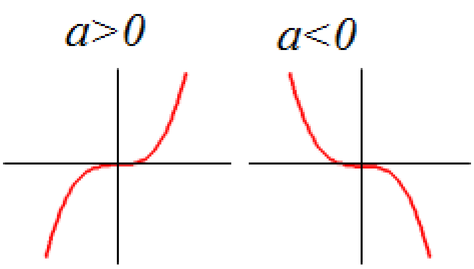
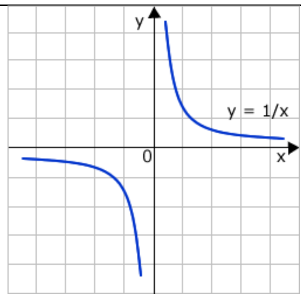
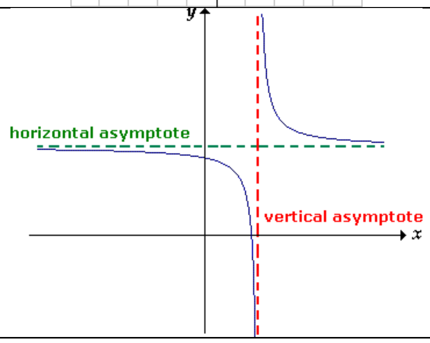
	$\{x \mid x \geq 7\}$ means ‘the set of all x’s, such that x is greater than or equal to 7’ The ‘x’ can be replaced by any letter. Some people use ‘:’ instead of ‘|’	 $\{x : -2 \leq x < 5\}$

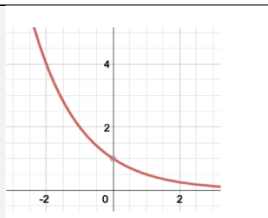
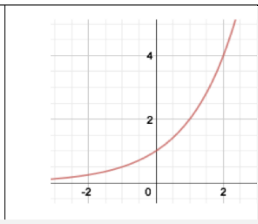
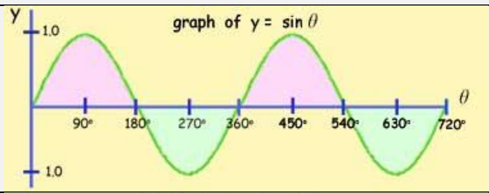
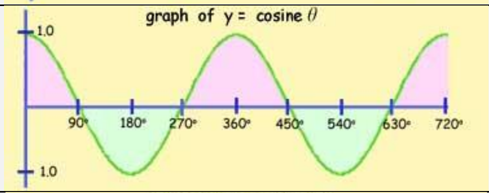
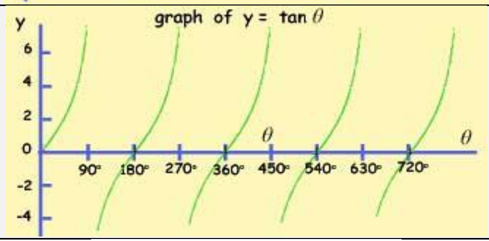
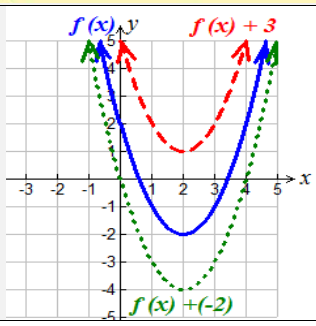
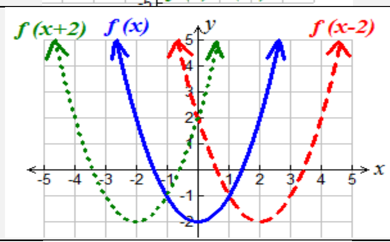
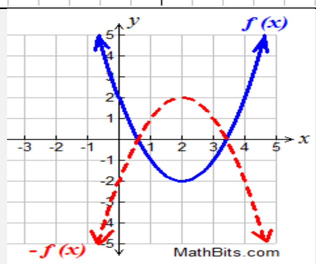
Subject: Maths **Topic: Proofs**

Topic/Skill	Definition/Tips	Example
1. Expression	A mathematical statement written using **symbols**, **numbers** or **letters**,	3x + 2 or $5y^2$
2. Equation	A statement showing that **two expressions are equal**	2y – 17 = 15
3. Identity	An equation that is **true for all values** of the variables An identity uses the symbol: ≡	$2x \equiv x+x$
4. Formula	Shows the **relationship** between **two or more variables**	Area of a rectangle = length x width or A= LxW
5. Coefficient	A **number** used to **multiply** a **variable**. It is the number that comes before/in front of a letter.	6z 6 is the coefficient z is the variable
6. Odds and Evens	An **even** number is a **multiple of 2** An **odd** number is an integer which is **not a multiple of 2**.	If n is an integer (whole number): An even number can be represented by **2n** or **2m** etc. An odd number can be represented by **2n-1** or **2n+1** or **2m+1** etc.
7. Consecutive Integers	Whole numbers that follow each other in order.	If n is an integer: **n, n+1, n+2** etc. are consecutive integers.
8. Square Terms	A term that is produced by multiply another term by itself.	If n is an integer: n^2, m^2 etc. are square integers
9. Sum	The sum of two or more numbers is the value you get when you add them together.	The sum of 4 and 6 is 10
10. Product	The product of two or more numbers is the value you get when you multiply them together.	The product of 4 and 6 is 24
11. Multiple	To show that an expression is a **multiple** of a number, you need to show that you can **factor out the number**.	$4n^2 + 8n - 12$ is a multiple of 4 because it can be written as: $4(n^2 + 2n - 3)$

Topic: Equation of a Circle and Tangent

Topic/Skill	Definition/Tips	Example
1. Equation of a Circle	The equation of a **circle**, **centre (0,0), radius r**, is: $$x^2 + y^2 = r^2$$	 $x^2 + y^2 = 25$
2. Tangent	A straight **line** that **touches** a circle at **exactly one point**, never entering the circle's interior. A **radius** is **perpendicular** to a **tangent** at the **point of contact**.	
3. Gradient	**Gradient** is another word for **slope**. $$\boldsymbol{G = \frac{Rise}{Run} = \frac{Change\ in\ y}{Change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}}$$	
4. Circle Theorem 5	**A tangent is perpendicular to the radius at the point of contact.** 	 $y = 5cm$ (Pythagoras' Theorem)

Topic/Skill	**Definition/Tips**	**Example**
1. Coordinates	Written in **pairs**. The **first** term is the **x-coordinate** (movement **across**). The **second** term is the **y-coordinate** (movement **up or down**)	 A: (4,7) B: (-6,-3)
2. Linear Graph	**Straight line** graph. The **equation** of a linear graph can contain an **x-term**, a **y-term** and a **number**.	Example:  Other examples: $x = y$ $y = 4$ $x = -2$ $y = 2x - 7$ $y + x = 10$ $2y - 4x = 12$
3. Quadratic Graph	A '**U-shaped**' curve called a **parabola**. The equation is of the form $y = \boldsymbol{ax^2 + bx + c}$, where a, b and c are numbers, $\boldsymbol{a \neq 0}$. If $\boldsymbol{a < 0,}$ the parabola is **upside down**.	
4. Cubic Graph	The equation is of the form $\boldsymbol{y = ax^3 + k}$, where $\boldsymbol{k}$ **is a number**. If $\boldsymbol{a > 0}$, the curve is **increasing**. If $\boldsymbol{a < 0}$, the curve is **decreasing**.	
5. Reciprocal Graph	The equation is of the form $\boldsymbol{y = \frac{A}{x}}$, where $\boldsymbol{A}$ **is a number** and $\boldsymbol{x \neq 0}$. The graph has **asymptotes** on the **x-axis and y-axis**.	
6. Asymptote	A **straight line** that a graph **approaches** but **never touches**.	

7. Exponential Graph	The equation is of the form $\boldsymbol{y = a^x}$, where a is a number called the **base**. If $\boldsymbol{a > 1}$ the graph **increases**. If $\boldsymbol{0 < a < 1}$, the graph **decreases**. The graph has an **asymptote** which is the **x-axis**.	 
8. $y = \sin x$	Key Coordinates: $(\mathbf{0},\mathbf{0}),(\mathbf{90},\mathbf{1}),(\mathbf{180},\mathbf{0}),(\mathbf{270},-\mathbf{1}),(\mathbf{360},\mathbf{0}$ y is never more than 1 or less than -1. Pattern repeats every 360°.	
9. $y = \cos x$	Key Coordinates: $(\mathbf{0},\mathbf{1}),(\mathbf{90},\mathbf{0}),(\mathbf{180},-\mathbf{1}),(\mathbf{270},\mathbf{0}),(\mathbf{360},\mathbf{1}$ y is never more than 1 or less than -1. Pattern repeats every 360°.	
10. $y = \tan x$	Key Coordinates: $(\mathbf{0},\mathbf{0}),(\mathbf{45},\mathbf{1}),(\mathbf{135},-\mathbf{1}),(\mathbf{180},\mathbf{0}),$ $(\mathbf{225},\mathbf{1}),(\mathbf{315},-\mathbf{1}),(\mathbf{360},\mathbf{0})$ **Asymptotes** at $\boldsymbol{x = 90}$ and $\boldsymbol{x = 270}$ Pattern repeats every 360°.	
11. $f(x) + a$	**Vertical translation** up a units. $\binom{0}{a}$	
12. $f(x + a)$	**Horizontal translation** <u>left</u> a units. $\binom{-a}{0}$	
13. $-f(x)$	**Reflection** over the **x-axis**.	
14. $f(-x)$	**Reflection** over the **y-axis**.