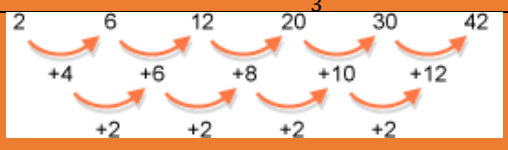
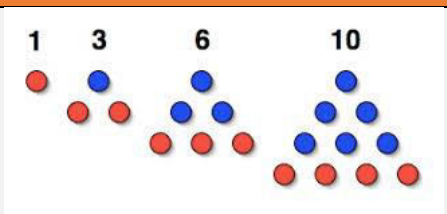
Topic:
Algebra

Topic/Skill	Definition/Tips	Example
1. Expression	A mathematical statement written using **symbols**, **numbers** or **letters**,	$3x + 2$ or $5y^2$
2. Equation	A statement showing that **two expressions are equal**	$2y - 17 = 15$
3. Identity	An equation that is **true for all values** of the variables An identity uses the symbol: ≡	$2x \equiv x+x$
4. Formula	Shows the **relationship** between **two or more variables**	Area of a rectangle = length x width or A= LxW
5. Simplifying Expressions	**Collect 'like terms'.** Be careful with negatives. x^2 and x are not like terms.	$2x + 3y + 4x - 5y + 3$ $= 6x - 2y + 3$ $3x + 4 - x^2 + 2x - 1 = 5x - x^2 + 3$
6. x times x	The answer is x^2 not $2x$.	Squaring is multiplying by itself, not by 2.
7. $p \times p \times p$	The answer is p^3 not $3p$	If p=2, then p^3=2x2x2=8, not 2x3=6
8. $p + p + p$	The answer is 3p not p^3	If p=2, then 2+2+2=6, not $2^3 = 8$
9. Expand	To expand a bracket, **multiply** each term **in the bracket** by the expression **outside** the bracket.	$3(m + 7) = 3x + 21$
10. Factorise	The **reverse** of **expanding**. Factorising is writing an expression as a product of terms by '**taking out' a common factor**.	$6x - 15 = 3(2x - 5)$, where 3 is the common factor.

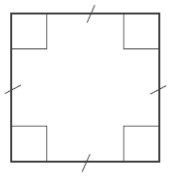
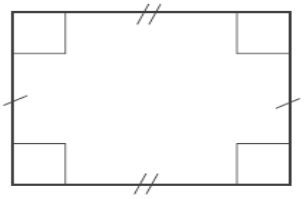
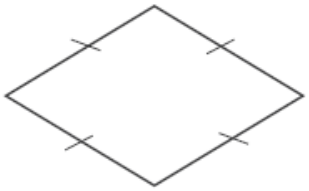
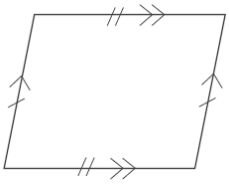
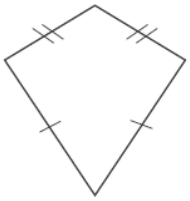
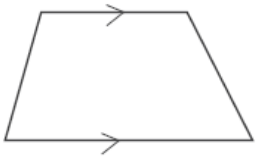
Topic/Skill	Definition/Tips	Example
1. Solve	To find the **answer**/value of something **Use inverse operations** on both sides of the equation (balancing method) until you find the value for the letter.	Solve $2x - 3 = 7$ Add 3 on both sides $2x = 10$ Divide by 2 on both sides $x = 5$
2. Inverse	**Opposite**	The inverse of addition is subtraction. The inverse of multiplication is division.
3. Rearranging Formulae	**Use inverse operations** on both sides of the formula (balancing method) until you find the expression for the letter.	Make x the subject of $y = \frac{2x-1}{z}$ Multiply both sides by z $yz = 2x - 1$ Add 1 to both sides $yz + 1 = 2x$ Divide by 2 on both sides $\frac{yz+1}{2} = x$ We now have x as the subject.
4. Writing Formulae	**Substitute letters for words** in the question.	Bob charges £3 per window and a £5 call out charge. $C = 3N + 5$ Where N=number of windows and C=cost
5. Substitution	**Replace letters with numbers**. Be careful of $5x^2$. You need to square first, then multiply by 5.	$a = 3, b = 2$ *and* $c = 5$. Find: 1. $2a = 2 \times 3 = 6$ 2. $3a - 2b = 3 \times 3 - 2 \times 2 = 5$ 3. $7b^2 - 5 = 7 \times 2^2 - 5 = 23$

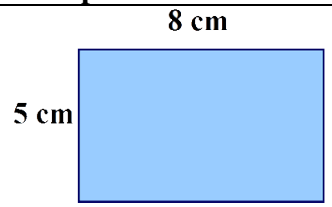
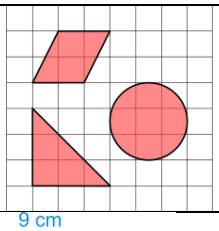
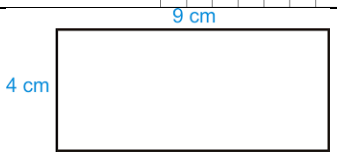
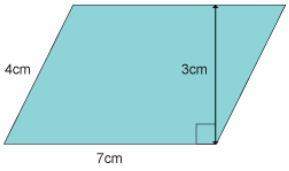
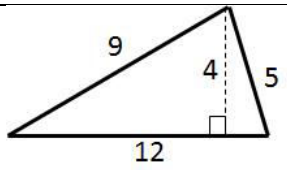
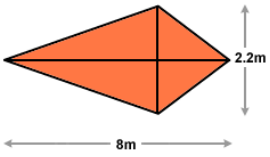
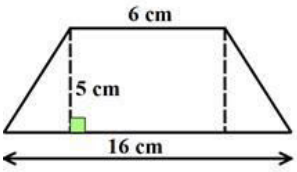
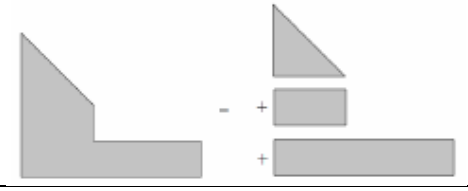
Topic/Skill	Definition/Tips	Example
1. Iteration	The act of **repeating a process** over and over again, often with the aim of **approximating** a desired result more closely. **Recursive** Notation: $x_{n+1} = \sqrt{3x_n + 6}$	$x_1 = 4$ $x_2 = \sqrt{3 \times 4 + 6} = 4.242640\ldots$ $x_3 = \sqrt{3 \times 4.242640\ldots + 6}$ $= 4.357576\ldots$
2. Iterative Method	To create an iterative formula, **rearrange** an equation with more than one x term to **make one of the x terms the subject**. You will be given the first value to substitute in, often called $\boldsymbol{x_1}$. **Keep substituting in your previous answer** until your answers are the same to a certain degree of accuracy. This is called converging to a limit. Use the 'ANS' button on your calculator to keep substituting in the previous answer.	Use an iterative formula to find the positive root of $x^2 - 3x - 6 = 0$ to 3 decimal places. $x_1 = 4$ Answer: $x^2 = 3x + 6$ $x = \sqrt{3x + 6}$ So $x_{n+1} = \sqrt{3x_n + 6}$ $x_1 = 4$ $x_2 = \sqrt{3 \times 4 + 6} = 4.242640\ldots$ $x_3 = \sqrt{3 \times 4.242640\ldots + 6}$ $= 4.357576\ldots$ Keep repeating… $x_7 = 4.372068.. = 4.372\ (3dp)$ $x_8 = 4.372208\ldots = 4.372\ (3dp)$ So answer is $x = 4.372\ (3dp)$

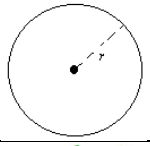
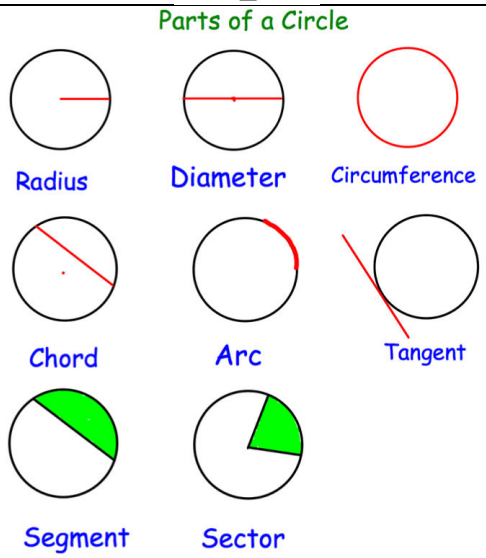
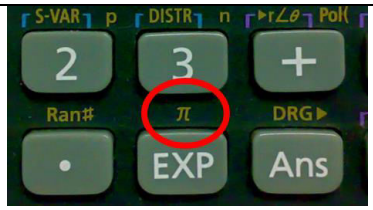
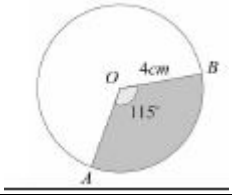
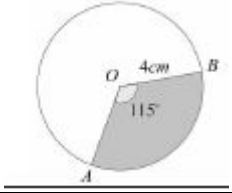
Topic/Skill	Definition/Tips	Example
1. Linear Sequence	A number pattern with a **common difference**.	2, 5, 8, 11… is a linear sequence
2. Term	**Each value** in a sequence is called a term.	In the sequence 2, 5, 8, 11…, 8 is the third term of the sequence.
3. Term-to-term rule	A rule which allows you to **find the next term** in a sequence if you **know the previous term**.	First term is 2. Term-to-term rule is 'add 3' Sequence is: 2, 5, 8, 11…
4. nth term	A rule which allows you to **calculate the term** that is in the **nth position** of the sequence. Also known as the 'position-to-term' rule. **n** refers to the **position** of a term in a sequence.	nth term is $3n - 1$ The 100^{th} term is $3 \times 100 - 1 = 299$
5. Finding the nth term of a linear sequence	1. Find the **difference**. 2. **Multiply that by n.** 3. Substitute $n = 1$ to **find out what number you need to add or subtract to get the first number in the sequence**.	Find the nth term of: 3, 7, 11, 15… 1. Difference is +4 2. Start with $4n$ 3. $4 \times 1 = 4$, so we need to subtract 1 to get 3. nth term = $4n - 1$
6. Fibonacci type sequences	A sequence where the next number is found by **adding up the previous two terms**	The Fibonacci sequence is: 1,1,2,3,5,8,13,21,34 ... An example of a Fibonacci-type sequence is: $4, 7, 11, 18, 29$ ...
7. Geometric Sequence	A sequence of numbers where each term is found by **multiplying the previous one** by a number called the **common ratio, r**.	An example of a geometric sequence is: $2, 10, 50, 250$ ... The common ratio is 5 Another example of a geometric sequence is: $81, -27, 9, -3, 1$ ... The common ratio is $-\frac{1}{3}$
8. Quadratic Sequence	A sequence of numbers where the **second difference is constant**. A quadratic sequence will have a n^2 term.	
9. nth term of a geometric sequence	ar^{n-1} where a is the first term and r is the common ratio	The nth term of $2, 10, 50, 250$ .... Is $2 \times 5^{n-1}$

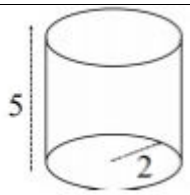
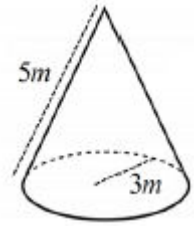
10. nth term of a quadratic sequence	1. Find the first and second differences. 2. Halve the second difference and multiply this by n^2. 3. Substitute $n = 1,2,3,4$ ... into your expression so far. 4. Subtract this set of numbers from the corresponding terms in the sequence from the question. 5. Find the nth term of this set of numbers. 6. Combine the nth terms to find the overall nth term of the quadratic sequence. Substitute values in to check your nth term works for the sequence.	Find the nth term of: 4, 7, 14, 25, 40.. Answer: Second difference = +4 → nth term = $2n^2$ Sequence: 4, 7, 14, 25, 40 $2n^2$ 2, 8, 18, 32, 50 Difference: 2, -1, -4, -7, -10 Nth term of this set of numbers is $-3n + 5$ Overall nth term: $2n^2 - 3n + 5$
11. Triangular numbers	The sequence which comes from a pattern of dots that form a triangle. $1, 3, 6, 10, 15, 21$ ...	

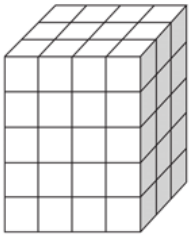
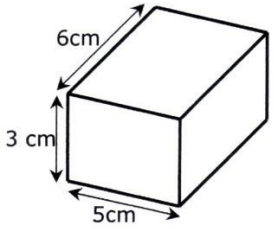
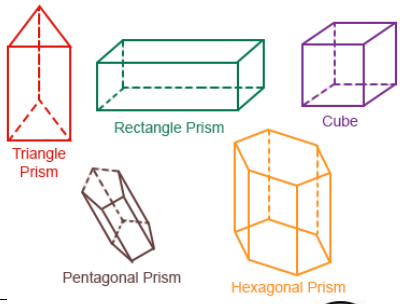
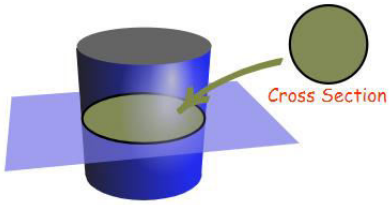
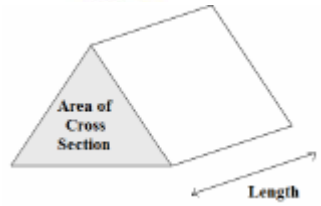
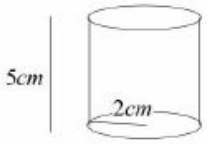
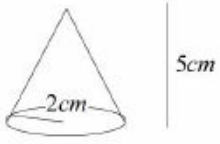
Higher Only Topics

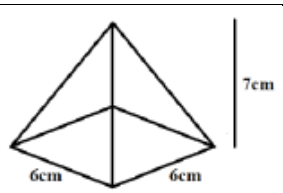
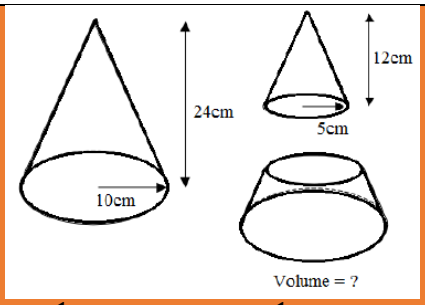
Topic/Skill	Definition/Tips	Example
1. Square	• **Four equal sides** • **Four right angles** • **Opposite sides parallel** • **Diagonals bisect** each other at **right angles** • **Four lines** of **symmetry** • **Rotational symmetry** of **order four**	
2. Rectangle	• **Two pairs** of **equal sides** • **Four right angles** • **Opposite sides parallel** • **Diagonals bisect** each other**, not at right angles** • **Two lines** of **symmetry** • **Rotational symmetry** of **order two**	
3. Rhombus	• **Four equal sides** • **Diagonally opposite angles are equal** • **Opposite sides parallel** • **Diagonals bisect** each other at **right angles** • **Two lines** of **symmetry** • **Rotational symmetry** of **order two**	
4. Parallelogram	• **Two pairs** of **equal sides** • **Diagonally opposite angles are equal** • **Opposite sides parallel** • **Diagonals bisect** each other**, not at right angles** • **No lines** of **symmetry** • **Rotational symmetry** of **order two**	
5. Kite	• **Two pairs** of **adjacent sides** of **equal** length • **One pair** of **diagonally opposite angles are equal** (where different length sides meet) • **Diagonals intersect** at **right angles, but do not bisect** • **One line** of **symmetry** • **No rotational symmetry**	
6. Trapezium	• **One pair** of **parallel sides** • **No lines of symmetry** • **No rotational symmetry** Special Case: Isosceles Trapeziums have one line of symmetry.	

Topic/Skill	Definition/Tips	Example
1. Perimeter	The **total distance** around the **outside** of a shape. Units include: mm, cm, m etc.	 $P = 8 + 5 + 8 + 5 = 26cm$
2. Area	The amount of **space inside** a shape. Units include: mm^2, cm^2, m^2	
3. Area of a Rectangle	**Length x Width**	 $A = 36cm^2$
4. Area of a Parallelogram	**Base x Perpendicular Height** Not the slant height.	 $A = 21cm^2$
5. Area of a Triangle	**Base x Height ÷ 2**	 $A = 24cm^2$
6. Area of a Kite	Split in to **two triangles** and use the method above.	 $A = 8.8m^2$
7. Area of a Trapezium	$\frac{(a+b)}{2} \times h$ "Half the sum of the parallel side, times the height between them. That is how you calculate the area of a trapezium"	 $A = 55cm^2$
8. Compound Shape	A shape made up of a **combination of other known shapes** put together.	

Topic/Skill	Definition/Tips	Example
1. Circle	A circle is the locus of all points equidistant from a central point.	
2. Parts of a Circle	**Radius** – the **distance** from the **centre** of a circle to the **edge** **Diameter** – the total **distance** across the **width** of a circle **through the centre**. **Circumference** – the **total distance** around the **outside** of a circle **Chord** – a **straight line** whose **end points lie on a circle** **Tangent** – a **straight line** which **touches** a circle at exactly **one point** **Arc** – a **part of the circumference** of a circle **Sector** – the **region** of a circle enclosed by **two radii** and their intercepted **arc** **Segment** – the **region** bounded by a **chord** and the **arc** created by the chord	
3. Area of a Circle	$\boldsymbol{A = \pi r^2}$ which means 'pi x radius squared'.	If the radius was 5cm, then: $A = \pi \times 5^2 = 78.5cm^2$
4. Circumference of a Circle	$\boldsymbol{C = \pi d}$ which means 'pi x diameter'	If the radius was 5cm, then: $C = \pi \times 10 = 31.4cm$
5. π ('pi')	Pi is the circumference of a circle divided by the diameter. $\boldsymbol{\pi \approx 3.14}$	
6. Arc Length of a Sector	The arc length is part of the circumference. Take the **angle** given **as a fraction over 360°** and **multiply** by the **circumference**.	Arc Length $= \frac{115}{360} \times \pi \times 8 = 8.03cm$ 
7. Area of a Sector	The area of a sector is part of the total area. Take the **angle** given **as a fraction over 360°** and **multiply** by the **area**.	Area $= \frac{115}{360} \times \pi \times 4^2 = 16.1cm^2$ 

8. Surface Area of a Cylinder	**Curved Surface Area = $\boldsymbol{\pi dh}$** or **$\boldsymbol{2\pi rh}$** **Total SA = $\boldsymbol{2\pi r^2 + \pi dh}$** or **$\boldsymbol{2\pi r^2 + 2\pi rh}$**	 $Total\ SA = 2\pi(2)^2 + \pi(4)(5) = 28\pi$
9. Surface Area of a Cone	**Curved Surface Area = $\boldsymbol{\pi rl}$** where $l = slant\ height$ **Total SA = $\boldsymbol{\pi rl + \pi r^2}$** You may need to use Pythagoras' Theorem to find the slant height	 $Total\ SA = \pi(3)(5) + \pi(3)^2 = 24\pi$
10. Surface Area of a Sphere	$\boldsymbol{SA = 4\pi r^2}$ Look out for hemispheres – halve the SA of a sphere and add on a circle (πr^2)	Find the surface area of a sphere with radius 3cm. $SA = 4\pi(3)^2 = 36\pi cm^2$

Topic/Skill	Definition/Tips	Example
1. Volume	Volume is a measure of the amount of space inside a solid shape. Units: mm^3, cm^3, m^3 etc.	
2. Volume of a Cube/Cuboid	$\boldsymbol{V = Length \times Width \times Height}$ $\boldsymbol{V = L \times W \times H}$ You can also use the Volume of a Prism formula for a cube/cuboid.	 volume = 6 x 5 x 3 = 90 cm^3
3. Prism	A prism is a 3D shape whose **cross section is the same** throughout.	
4. Cross Section	The **cross section** is the **shape** that **continues** all the way **through the prism**.	
5. Volume of a Prism	$\boldsymbol{V = Area\ of\ Cross\ Section \times Length}$ $\boldsymbol{V = A \times L}$	
6. Volume of a Cylinder	$\boldsymbol{V = \pi r^2 h}$	 $V = \pi(4)(5)$ $= 62.8cm^3$
7. Volume of a Cone	$\boldsymbol{V = \frac{1}{3}\pi r^2 h}$	 $V = \frac{1}{3}\pi(4)(5)$ $= 20.9cm^3$

8. Volume of a Pyramid	$\boldsymbol{Volume = \frac{1}{3}Bh}$ where B = area of the base	 $V = \frac{1}{3} \times 6 \times 6 \times 7 = 84cm^3$
9. Volume of a Sphere	$\boldsymbol{V = \frac{4}{3}\pi r^3}$ Look out for hemispheres – just halve the volume of a sphere.	Find the volume of a sphere with diameter 10cm. $V = \frac{4}{3}\pi(5)^3 = \frac{500\pi}{3}cm^3$
10. Frustums	A frustum is a solid (usually a cone or pyramid) with the **top removed**. Find the volume of the whole shape, then take away the volume of the small cone/pyramid removed at the top.	 $V = \frac{1}{3}\pi(10)^2(24) - \frac{1}{3}\pi(5)^2(12)$ $= 700\pi cm^3$

Higher Only Topics