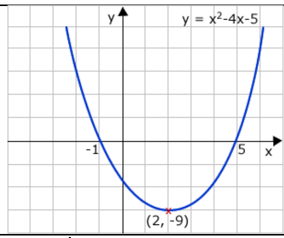
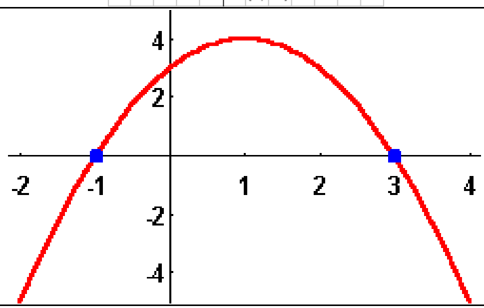
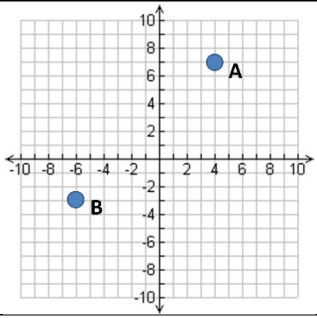
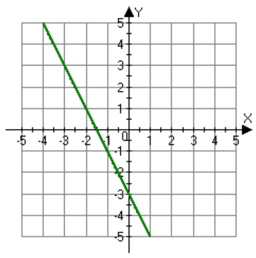
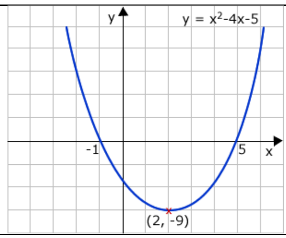
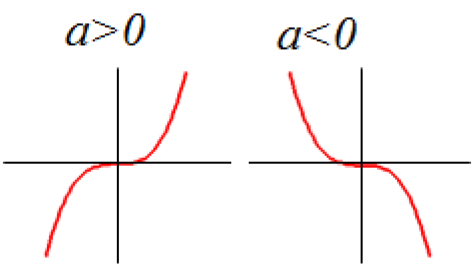
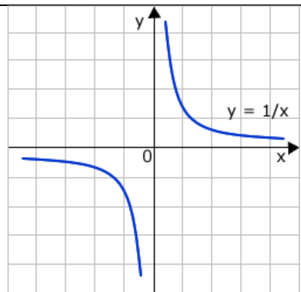
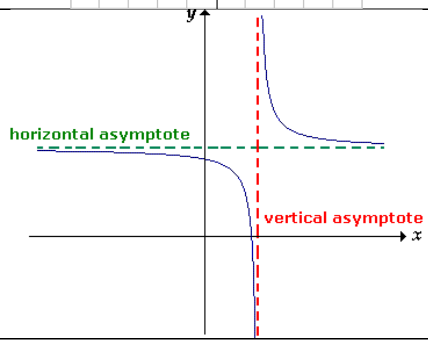
Topic: Further Quadratics

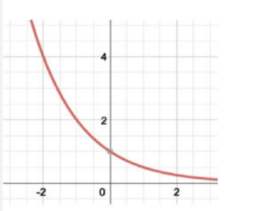
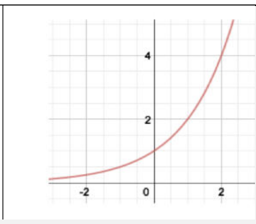
Topic/Skill	Definition/Tips	Example
1. Quadratic	A quadratic expression is of the form $$\boldsymbol{ax^2 + bx + c}$$ where a, b and c are numbers, $\boldsymbol{a \neq 0}$	Examples of quadratic expressions: x^2 $8x^2 - 3x + 7$ Examples of non-quadratic expressions: $2x^3 - 5x^2$ $9x - 1$
2. Factorising Quadratics	When a quadratic expression is in the form $x^2 + bx + c$ find the two numbers that **add to give b** and **multiply to give c**.	$x^2 + 7x + 10 = (x + 5)(x + 2)$ (because 5 and 2 add to give 7 and multiply to give 10) $x^2 + 2x - 8 = (x + 4)(x - 2)$ (because +4 and -2 add to give +2 and multiply to give -8)
3. Difference of Two Squares	An expression of the form $\boldsymbol{a^2 - b^2}$ can be factorised to give $(\boldsymbol{a + b})(\boldsymbol{a - b})$	$x^2 - 25 = (x + 5)(x - 5)$ $16x^2 - 81 = (4x + 9)(4x - 9)$
4. Solving Quadratics ($ax^2 = b$)	Isolate the x^2 term and square root both sides. Remember there will be a **positive and a negative solution**.	$2x^2 = 98$ $x^2 = 49$ $x = \pm 7$
5. Solving Quadratics ($ax^2 + bx = 0$)	**Factorise** and then **solve = 0**.	$x^2 - 3x = 0$ $x(x - 3) = 0$ $x = 0 \text{ or } x = 3$
6. Solving Quadratics by Factorising ($a = 1$)	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $x^2 + 3x - 10 = 0$ Factorise: $(x + 5)(x - 2) = 0$ $x = -5 \text{ or } x = 2$
7. Quadratic Graph	A **'U-shaped'** curve called a **parabola**. The equation is of the form $y = \boldsymbol{ax^2 + bx + c}$, where a, b and c are numbers, $\boldsymbol{a \neq 0}$. If $\boldsymbol{a < 0}$**,** the parabola is **upside down**.	
8. Roots of a Quadratic	A root is a **solution**. The roots of a quadratic are the ***x*-intercepts of the quadratic graph**.	

9. Turning Point of a Quadratic	A turning point is the **point where a quadratic turns**. On a **positive parabola**, the turning point is called a **minimum**. On a **negative parabola**, the turning point is called a **maximum**.	
10. Factorising Quadratics when $a \neq 1$	When a quadratic is in the form $ax^2 + bx + c$ 1. Multiply a by c = ac 2. Find two numbers that add to give b and multiply to give ac. 3. Re-write the quadratic, replacing bx with the two numbers you found. 4. Factorise in pairs – you should get the same bracket twice 5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets.	Factorise $6x^2 + 5x - 4$ 1. $6 \times -4 = -24$ 2. Two numbers that add to give +5 and multiply to give -24 are +8 and -3 3. $6x^2 + 8x - 3x - 4$ 4. Factorise in pairs: $2x(3x + 4) - 1(3x + 4)$ 5. Answer = $(3x + 4)(2x - 1)$
11. Solving Quadratics by Factorising $(a \neq 1)$	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $2x^2 + 7x - 4 = 0$ Factorise: $(2x - 1)(x + 4) = 0$ $x = \frac{1}{2}$ or $x = -4$

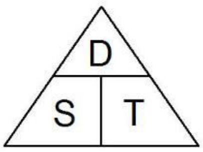
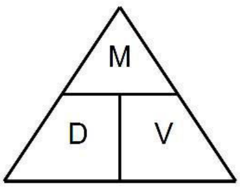
Subject: Maths

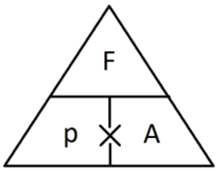
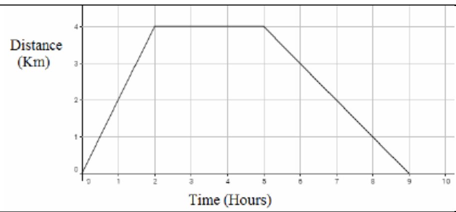
Topic: Graphs and Graph Transformations

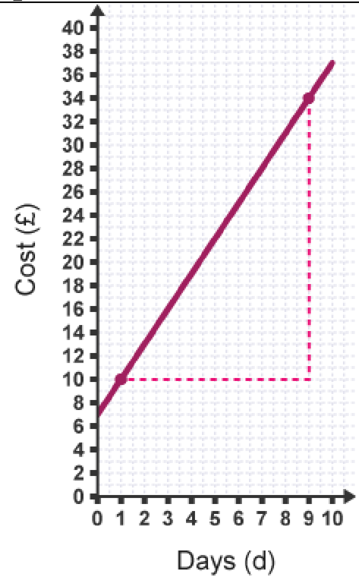
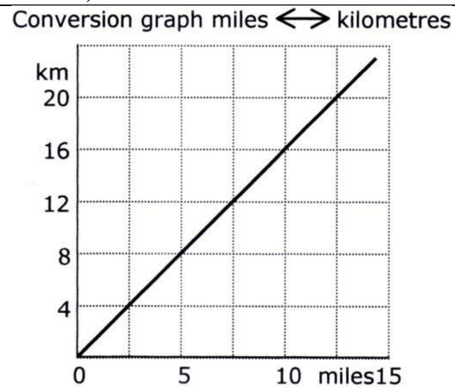
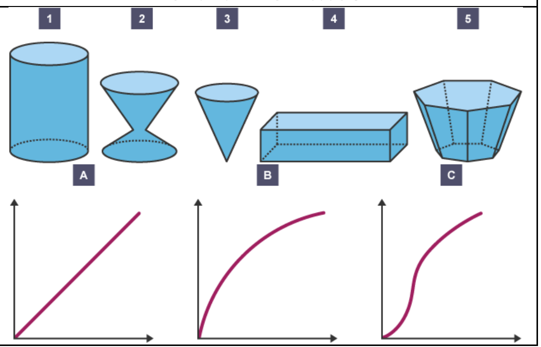
Topic/Skill	Definition/Tips	Example
1. Coordinates	Written in **pairs**. The **first** term is the **x-coordinate** (movement **across**). The **second** term is the **y-coordinate** (movement **up or down**)	 A: (4,7) B: (-6,-3)
2. Linear Graph	**Straight line** graph. The **equation** of a linear graph can contain an **x-term**, a **y-term** and a **number**.	Example:  Other examples: $x = y$ $y = 4$ $x = -2$ $y = 2x - 7$ $y + x = 10$ $2y - 4x = 12$
3. Quadratic Graph	A '**U-shaped**' curve called a **parabola**. The equation is of the form $y = \boldsymbol{ax^2 + bx + c}$, where a, b and c are numbers, $\boldsymbol{a \neq 0}$. If $\boldsymbol{a < 0}$**,** the parabola is **upside down**.	
4. Cubic Graph	The equation is of the form $\boldsymbol{y = ax^3 + k}$, where $\boldsymbol{k}$ **is an number**. If $\boldsymbol{a > 0}$, the curve is **increasing**. If $\boldsymbol{a < 0}$, the curve is **decreasing**.	
5. Reciprocal Graph	The equation is of the form $\boldsymbol{y = \frac{A}{x}}$, where $\boldsymbol{A}$ **is a number** and $\boldsymbol{x \neq 0}$. The graph has **asymptotes** on the **x-axis and y-axis**.	
6. Asymptote	A **straight line** that a graph **approaches** but **never touches**.	

7. Exponential Graph	The equation is of the form $\boldsymbol{y = a^x}$**,** where a is a number called the **base**. If $\boldsymbol{a > 1}$ the graph **increases**. If $\boldsymbol{0 < a < 1}$, the graph **decreases**. The graph has an **asymptote** which is the **x-axis**.		

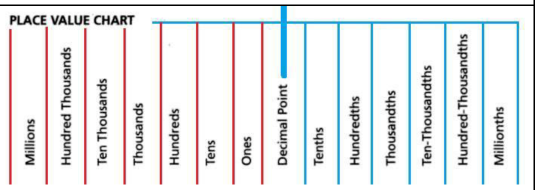
Topic: Compound Measures

Topic/Skill	Definition/Tips	Example
1. Metric System	A system of measures based on: - the metre for length - the kilogram for mass - the second for time **Length: mm, cm, m, km** **Mass: mg, g, kg** **Volume: ml, cl, l**	$1 kilometres = 1000\ metres$ $1\ metre = 100\ centimetres$ $1\ centimetre = 10\ millimetres$ $1\ kilogram = 1000\ grams$
2. Imperial System	A system of weights and measures originally developed in England, usually based on human quantities **Length: inch, foot, yard, miles** **Mass: lb, ounce, stone** **Volume: pint, gallon**	$1lb = 16\ ounces$ $1\ foot = 12\ inches$ $1\ gallon = 8\ pints$
3. Metric and Imperial Units	Use the **unitary method** to convert between metric and imperial units.	$5\ miles \approx 8\ kilometres$ $1\ gallon \approx 4.5\ litres$ $2.2\ pounds \approx 1\ kilogram$ $1\ inch = 2.5\ centimetres$
4. Speed, Distance, Time	**Speed = Distance ÷ Time** **Distance = Speed x Time** **Time = Distance ÷ Speed**  Remember the correct units.	Speed = 4mph Time = 2 hours Find the Distance. $D = S \times T = 4 \times 2 = 8\,miles$
5. Density, Mass, Volume	**Density = Mass ÷ Volume** **Mass = Density x Volume** **Volume = Mass ÷ Density**  Remember the correct units.	Density = 8kg/m^3 Mass = 2000g Find the Volume. $V = M \div D = 2 \div 8 = 0.25m^3$
6. Pressure, Force, Area	**Pressure = Force ÷ Area** **Force = Pressure x Area** **Area = Force ÷ Pressure**	Pressure = 10 Pascals Area = 6cm^2 Find the Force

	 Remember the correct units.	$F = P \times A = 10 \times 6 = 60\ N$
7. Distance-Time Graphs	You can find the **speed** from the **gradient** of the line (Distance ÷ Time) The steeper the line, the quicker the speed. A **horizontal** line means the object is not moving (**stationary**).	

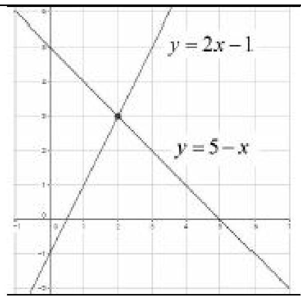
Topic/Skill	Definition/Tips	Example
1. Real Life Graphs	Graphs that are supposed to model some real-life situation. The actual meaning of the values depends on the labels and units on each axis. The **gradient** might have a contextual meaning. The **y-intercept** might have a contextual meaning. The **area** under the graph might have a contextual meaning.	 A graph showing the cost of hiring a ladder for various numbers of days. The gradient shows the cost per day. It costs £3/day to hire the ladder. The y-intercept shows the additional cost/deposit/fixed charge (something not linked to how long the ladder is hired for). The additional cost is £7.
2. Conversion Graph	A line graph to **convert one unit to another**. Can be used to convert units (eg. miles and kilometres) or currencies ($ and £) Find the value you know on one axis, read up/across to the conversion line and read the equivalent value from the other axis.	 $8\ km = 5\ miles$
3. Depth of Water in Containers	Graphs can be used to show how the depth of water changes as different shaped containers are filled with water at a constant rate.	

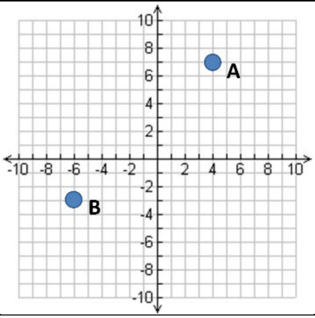
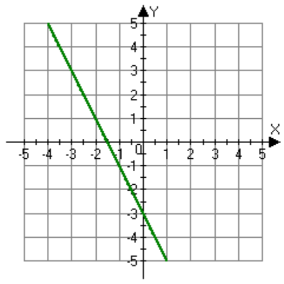
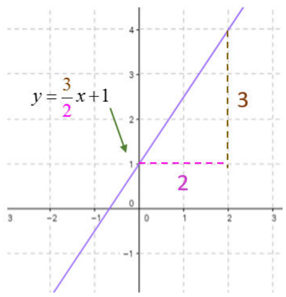
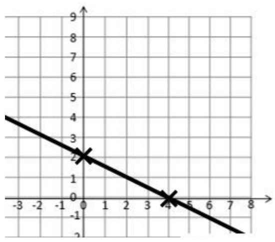
Topic: Accuracy

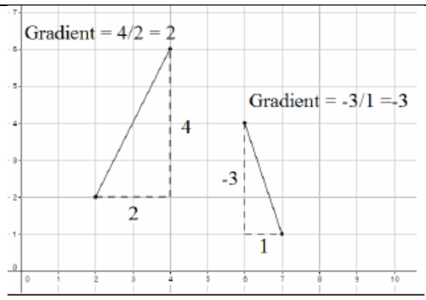
Topic/Skill	Definition/Tips	Example
1. Place Value	The **value** of where a **digit** is within a number.	In 726, the value of the 2 is 20, as it is in the 'tens' column.
2. Place Value Columns	The names of the columns that **determine the value of each digit**. The 'ones' column is also known as the 'units' column.	
3. Rounding	To make a number simpler but keep its value close to what it was. If the **digit to the right** of the rounding digit is **less than 5, round down**. If the **digit to the right** of the rounding digit is **5 or more, round up**.	74 rounded to the nearest ten is 70, because 74 is closer to 70 than 80. 152,879 rounded to the nearest thousand is 153,000.
4. Decimal Place	The **position** of a digit to the **right of a decimal point**.	In the number 0.372, the 7 is in the second decimal place. 0.372 rounded to two decimal places is 0.37, because the 2 tells us to round down. Careful with money - don't write £27.4, instead write £27.40
5. Significant Figure	The significant figures of a number are the digits which **carry meaning** (ie. are significant) to the size of the number. The **first significant figure** of a number **cannot be zero**. In a number with a decimal, trailing zeros are not significant.	In the number 0.00821, the first significant figure is the 8. In the number 2.740, the 0 is not a significant figure. 0.00821 rounded to 2 significant figures is 0.0082. 19357 rounded to 3 significant figures is 19400. We need to include the two zeros at the end to keep the digits in the same place value columns.
6. Truncation	A method of approximating a decimal number by **dropping all decimal places** past a certain point **without rounding**.	3.14159265… can be truncated to 3.1415 (note that if it had been rounded, it would become 3.1416)
7. Error Interval	A **range of values** that a number could have taken before being rounded or truncated. An error interval is written using inequalities, with a **lower bound** and an **upper bound**. Note that the lower bound inequality can be 'equal to', but the upper bound cannot be	0.6 has been rounded to 1 decimal place. The error interval is: $0.55 \leq x < 0.65$ The lower bound is 0.55 The upper bound is 0.65

	‘equal to’.	
8. Estimate	To find something **close to the correct answer**.	An estimate for the height of a man is 1.8 metres.
9. Approximation	When using approximations to estimate the solution to a calculation, **round each number in the calculation to 1 significant figure**. ≈ means ‘approximately equal to’	$\frac{348+692}{0.526} \approx \frac{300+700}{0.5} = 2000$ ‘Note that dividing by 0.5 is the same as multiplying by 2’

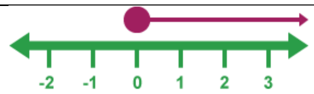
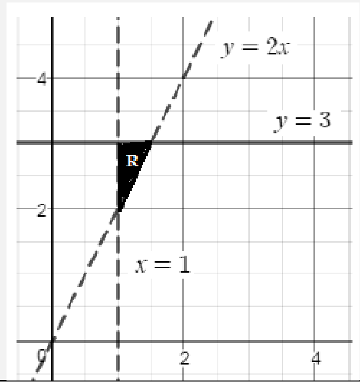
Topic/Skill	Definition/Tips	Example
1. Simultaneous Equations	A set of **two or more equations**, each involving **two or more variables** (letters). The **solutions** to simultaneous equations **satisfy both**/all of the **equations**.	$2x + y = 7$ $3x - y = 8$ $x = 3$ $y = 1$
2. Variable	A **symbol**, usually a **letter**, which **represents a number** which is usually unknown.	In the equation $x + 2 = 5$, x is the variable.
3. Coefficient	A **number** used to **multiply** a **variable**. It is the number that comes before/in front of a letter.	6z 6 is the coefficient z is the variable
4. Solving Simultaneous Equations (by Elimination)	1. **Balance** the **coefficients** of one of the variables. 2. **Eliminate** this variable by adding or subtracting the equations (**Same Sign Subtract, Different Sign Add**) 3. **Solve** the linear equation you get using the other variable. 4. **Substitute** the value you found back into one of the previous equations. 5. **Solve** the equation you get. 6. **Check** that the two values you get satisfy both of the original equations.	$5x + 2y = 9$ $10x + 3y = 16$ Multiply the first equation by 2. $10x + 4y = 18$ $10x + 3y = 16$ Same Sign Subtract (+10x on both) $y = 2$ Substitute $y = 2$ in to equation. $5x + 2 \times 2 = 9$ $5x + 4 = 9$ $5x = 5$ $x = 1$ Solution: $x = 1, y = 2$
5. Solving Simultaneous Equations (by Substitution)	1. **Rearrange** one of the equations into the form $y =$... or $x =$... 2. **Substitute** the right-hand side of the rearranged equation into the other equation. 3. Expand and **solve** this equation. 4. **Substitute** the value into the $y =$... or $x =$... equation. 5. **Check** that the two values you get satisfy both of the original equations.	$y - 2x = 3$ $3x + 4y = 1$ Rearrange: $y - 2x = 3 \rightarrow y = 2x + 3$ Substitute: $3x + 4(2x + 3) = 1$ Solve: $3x + 8x + 12 = 1$ $11x = -11$ $x = -1$ Substitute: $y = 2 \times -1 + 3$ $y = 1$ Solution: $x = -1, y = 1$

6. Solving Simultaneous Equations (Graphically)	**Draw the graphs** of the two equations. The **solutions** will be **where the lines meet**. The solution can be written as a **coordinate**.	 $y = 5 - x$ and $y = 2x - 1$. They meet at the point with coordinates (2,3) so the answer is $x = 2$ and $y = 3$

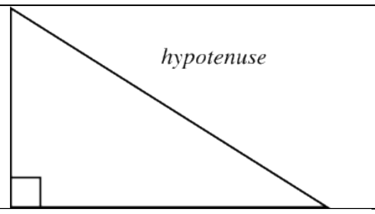
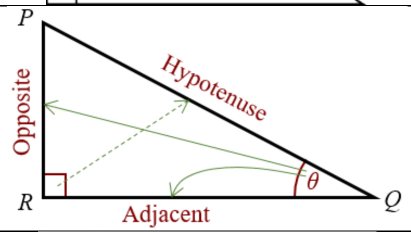
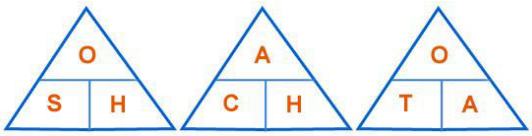
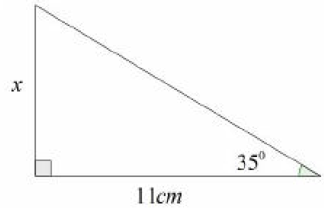
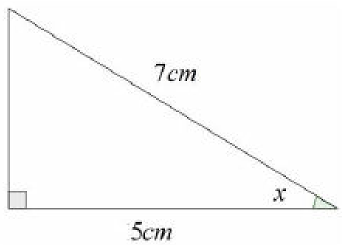
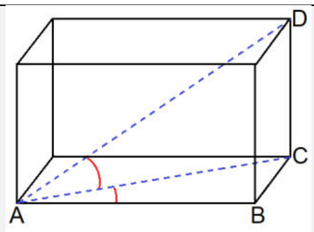
Topic/Skill	Definition/Tips	Example
1. Coordinates	Written in **pairs**. The **first** term is the **x-coordinate** (movement **across**). The **second** term is the **y-coordinate** (movement **up or down**)	 A: (4,7) B: (-6,-3)
2. Midpoint of a Line	Method 1: **add the x coordinates and divide by 2**, **add the y coordinates and divide by 2** Method 2: Sketch the line and find the values half way between the two x and two y values.	Find the midpoint between (2,1) and (6,9) $\frac{2+6}{2} = 4$ and $\frac{1+9}{2} = 5$ So, the midpoint is (4,5)
3. Linear Graph	**Straight line** graph. The general equation of a linear graph is $\boldsymbol{y = mx + c}$ where $\boldsymbol{m}$ **is the gradient** and c **is the y-intercept**. The **equation** of a linear graph can contain an **x-term**, a **y-term** and a **number**.	Example:  Other examples: $x = y$ $y = 4$ $x = -2$ $y = 2x - 7$ $y + x = 10$ $2y - 4x = 12$
4. Plotting Linear Graphs	Method 1: **Table of Values** Construct a table of values to calculate coordinates. Method 2: **Gradient-Intercept Method** (use when the equation is in the form $y = mx + c$) 1. Plots the y-intercept 2. Using the gradient, plot a second point. 3. Draw a line through the two points plotted. Method 3: **Cover-Up Method** (use when the equation is in the form $ax + by = c$) 1. Cover the x term and solve the resulting equation. Plot this on the $x - axis$. 2. Cover the y term and solve the resulting equation. Plot this on the $y - axis$. 3. Draw a line through the two points plotted.	**x**: -3, -2, -1, 0, 1, 2, 3 **y= x +3**: 0, 1, 2, 3, 4, 5, 6   $2x + 4y = 8$

5. Gradient	The gradient of a line is how **steep** it is. **Gradient =** $\dfrac{\textbf{Change in } \boldsymbol{y}}{\textbf{Change in } \boldsymbol{x}} = \dfrac{\textbf{Rise}}{\textbf{Run}}$ The gradient can be positive (sloping upwards) or negative (sloping downwards)	
6. Finding the Equation of a Line <u>given a point and a gradient</u>	**Substitute** in the **gradient (m)** and **point (x,y)** in to the equation $\boldsymbol{y = mx + c}$ and **solve for c**.	Find the equation of the line with gradient 4 passing through (2,7). $y = mx + c$ $7 = 4 \times 2 + c$ $c = -1$ $y = 4x - 1$
7. Finding the Equation of a Line <u>given two points</u>	Use the two points to **calculate the gradient**. Then **repeat the method above** using the gradient and either of the points.	Find the equation of the line passing through (6,11) and (2,3) $m = \dfrac{11 - 3}{6 - 2} = 2$ $y = mx + c$ $11 = 2 \times 6 + c$ $c = -1$ $y = 2x - 1$
8. Parallel Lines	If two lines are **parallel**, they will have the **same gradient**. The value of m will be the same for both lines.	Are the lines $y = 3x - 1$ and $2y - 6x + 10 = 0$ parallel? Answer: Rearrange the second equation in to the form $y = mx + c$ $2y - 6x + 10 = 0 \rightarrow y = 3x - 5$ Since the two gradients are equal (3), the lines are parallel.
9. Perpendicular Lines	If two lines are **perpendicular**, the **product** of their **gradients** will always equal **-1**. The gradient of one line will be the **negative reciprocal** of the gradient of the other line. You may need to rearrange equations of lines to compare gradients (they need to be in the form $y = mx + c$)	Find the equation of the line perpendicular to $y = 3x + 2$ which passes through (6,5) Answer: As they are perpendicular, the gradient of the new line will be $-\frac{1}{3}$ as this is the negative reciprocal of 3. $y = mx + c$

		$$5 = -\frac{1}{3} \times 6 + c$$ $$c = 7$$ $$y = -\frac{1}{3}x + 7$$ Or $$3x + x - 7 = 0$$

Topic/Skill	**Definition/Tips**	**Example**
1. Inequality	An inequality says that two values are **not equal**. $a \neq b$ means that a is not equal to b.	$7 \neq 3$ $x \neq 0$
2. Inequality symbols	$\boldsymbol{x > 2}$ means **x is greater than 2** $\boldsymbol{x < 3}$ means **x is less than 3** $\boldsymbol{x \geq 1}$ means **x is greater than or equal to 1** $\boldsymbol{x \leq 6}$ means **x is less than or equal to 6**	State the integers that satisfy $-2 < x \leq 4$. -1, 0, 1, 2, 3, 4
3. Inequalities on a Number Line	Inequalities can be shown on a number line. **Open circles** are used for numbers that are **less than or greater than** $(< or >)$ **Closed circles** are used for numbers that are **less than or equal or greater than or equal** $(\leq or \geq)$	 $x \geq 0$  $x < 2$  $-5 \leq x < 4$
4. Graphical Inequalities	Inequalities can be represented on a coordinate grid. If the inequality is **strict** $(x > 2)$ then use a **dotted line**. If the inequality is **not strict** $(x \leq 6)$ then use a **solid line**. **Shade** the **region** which satisfies all the inequalities.	Shade the region that satisfies: $y > 2x, x > 1$ and $y \leq 3$ 

Topic: Right Angled Trigonometry

Topic/Skill	Definition/Tips	Example
1. Trigonometry	The **study** of **triangles**.	
2. Hypotenuse	The **longest side** of a **right-angled triangle**. Is always **opposite** the **right angle**.	
3. Adjacent	**Next to**	
4. Trigonometric Formulae	Use **SOHCAHTOA**. $\boldsymbol{\sin\theta = \frac{O}{H}}$ $\boldsymbol{\cos\theta = \frac{A}{H}}$ $\boldsymbol{\tan\theta = \frac{O}{A}}$  When finding a missing angle, use the 'inverse' trigonometric function by pressing the 'shift' button on the calculator.	 Use 'Opposite' and 'Adjacent', so use 'tan' $\tan 35 = \frac{x}{11}$ $x = 11\tan 35 = 7.70cm$  Use 'Adjacent' and 'Hypotenuse', so use 'cos' $\cos x = \frac{5}{7}$ $x = cos^{-1}\left(\frac{5}{7}\right) = 44.4°$
5. 3D Trigonometry	Find missing lengths by **identifying right angled triangles**. You will often have to find a missing length you are not asked for before finding the missing length you are asked for.	

<table>
<tr><th>Topic/Skill</th><th>Definition/Tips</th><th>Example</th></tr>
<tr><td>1. Exact Values for Angles in Trigonometry</td><td>
<table>
<tr><th></th><th>0°</th><th>30°</th><th>45°</th><th>60°</th><th>90°</th></tr>
<tr><td>sin</td><td>0</td><td>$\frac{1}{2}$</td><td>$\frac{\sqrt{2}}{2}$</td><td>$\frac{\sqrt{3}}{2}$</td><td>1</td></tr>
<tr><td>cos</td><td>1</td><td>$\frac{\sqrt{3}}{2}$</td><td>$\frac{\sqrt{2}}{2}$</td><td>$\frac{1}{2}$</td><td>0</td></tr>
<tr><td>tan</td><td>0</td><td>$\frac{1}{\sqrt{3}}$</td><td>1</td><td>$\sqrt{3}$</td><td>----</td></tr>
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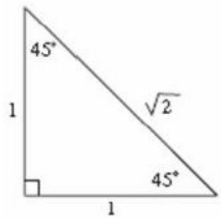

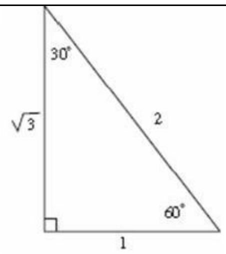

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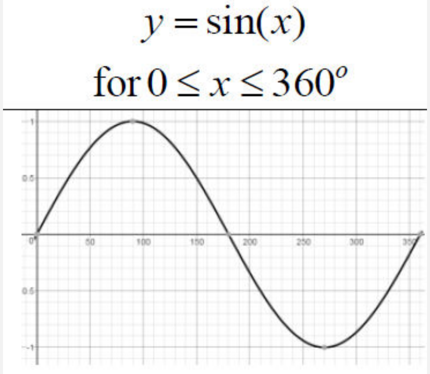
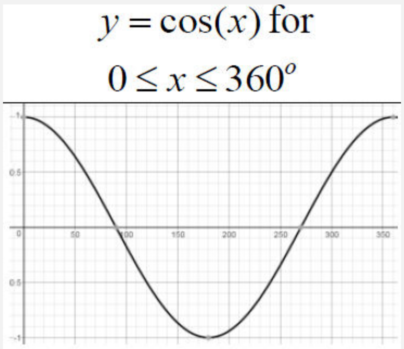
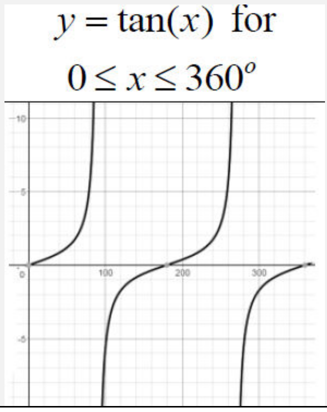
<tr><td>2. Graphs of Trigonometric Functions</td><td>






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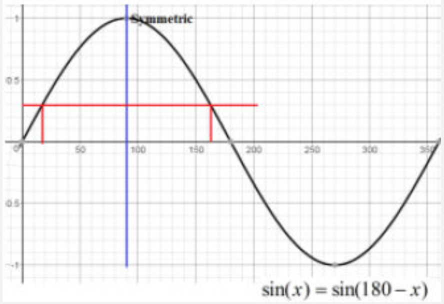

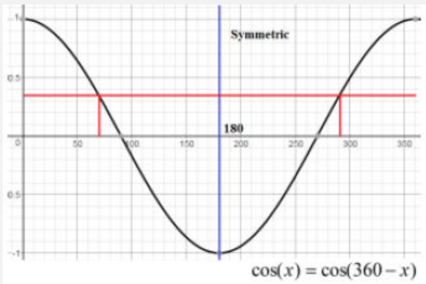

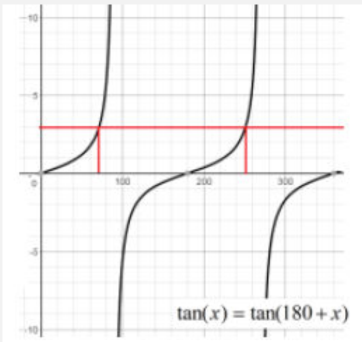

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Subject: Maths **Topic: Proofs**

Topic/Skill	Definition/Tips	Example
1. Expression	A mathematical statement written using **symbols**, **numbers** or **letters**,	$3x + 2$ or $5y^2$
2. Equation	A statement showing that **two expressions are equal**	$2y - 17 = 15$
3. Identity	An equation that is **true for all values** of the variables An identity uses the symbol: ≡	$2x \equiv x+x$
4. Formula	Shows the **relationship** between **two or more variables**	Area of a rectangle = length x width or A= LxW
5. Coefficient	A **number** used to **multiply** a **variable**. It is the number that comes before/in front of a letter.	6z 6 is the coefficient z is the variable
6. Odds and Evens	An **even** number is a **multiple of 2** An **odd** number is an integer which is **not a multiple of 2**.	If n is an integer (whole number): An even number can be represented by **2n** or **2m** etc. An odd number can be represented by **2n-1** or **2n+1** or **2m+1** etc.
7. Consecutive Integers	Whole numbers that follow each other in order.	If n is an integer: **n, n+1, n+2** etc. are consecutive integers.
8. Square Terms	A term that is produced by multiply another term by itself.	If n is an integer: n^2, m^2 etc. are square integers
9. Sum	The sum of two or more numbers is the value you get when you add them together.	The sum of 4 and 6 is 10
10. Product	The product of two or more numbers is the value you get when you multiply them together.	The product of 4 and 6 is 24
11. Multiple	To show that an expression is a **multiple** of a number, you need to show that you can **factor out the number**.	$4n^2 + 8n - 12$ is a multiple of 4 because it can be written as: $4(n^2 + 2n - 3)$